

## Modelling Leverett J Function Using NMR Data

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### Abstract

This paper is concerned essentially with understanding the Leverett J function as an expression of an intrinsic property of the pore space of rocks by which different rocks can be compared, irrespective of their bulk porosity and permeability. Since NMR data is the only type of data which comes closest to expressing the principal attributes of the pore space of rocks in a natural way, modelling the Leverett J function would be a productive way of attaining a quantitative physical understanding of the Leverett J function as an attribute of the pore structure of rocks. In this paper two approaches, (one of which is based on the fractal model of pore space) to the modelling of the Leverett J Function using NMR data are presented. Two ways of deriving a saturation height function are also demonstrated.

### 1.1 Introduction

Consider an intrusion experiment where non-wetting fluid replaces wetting fluid. Consider a stage of intrusion where the wetting fluid saturation is  $S_w$ , and the capillary pressure is  $P_C(S_w)$ . Let  $r_{threshold}(S_w)$  denote threshold radius for wetting fluid saturation of  $S_w$ . Let the interfacial tension and the angle of contact be denoted as  $\sigma, \theta$ , respectively.

Let  $V_p(r)$  denote the distribution of the incremental pore-volume over the value of the pore radius  $r$ .  $V_p(r)$  is defined by the equation,

$$dV_p(r, r + dr) = V_p(r)dr \quad (1)$$

Here,  $dV_p(r, r + dr)$  denotes the number of pores per unit rock volume, whose radius lies within the interval  $(r, r + dr)$ . Let  $r_{min}, r_{max}$  and  $\bar{r}$  respectively stand for the minimum pore radius the maximum pore radius and the mean pore radius for the assemblage of pores. Let  $\varphi$  denote the value of the pore volume per unit rock volume (porosity).

$$\bar{r} = \frac{\int_{r_{min}}^{r_{max}} r V_p(r) dr}{\int_{r_{min}}^{r_{max}} V_p(r) dr} = \frac{1}{\varphi} \int_{r_{min}}^{r_{max}} r V_p(r) dr \quad (2)$$

Let  $k$  denote the value of the permeability of the rock. The following relation can be shown to hold between  $\bar{r}, k, \varphi$ :

$$\bar{r} = C \sqrt{\frac{k}{\varphi}} \text{ where } C \text{ denotes the quantity } \left(\frac{1}{\sqrt{8}}\right) \text{ and } \tau \text{ is the hydraulic tortuosity of pore space.} \quad (3)$$

Let  $S_{wm}$  denote the wetting fluid saturation at that stage of intrusion, when the value of the threshold radius is  $\bar{r}$ .

We have, using Washburn's Equation (Washburn, 1921), and where  $\lambda$  stands for the representative value of the ratio of the pore radius to the pore throat radius:

$$P_C(S_w) = \lambda \frac{2\sigma \cos\theta}{r_{threshold}(S_w)} \quad (4)$$

$$P_C(S_{wm}) = \frac{2\sigma \cos\theta}{\bar{r}} \lambda = \frac{2\lambda \sigma \cos\theta}{C} \sqrt{\frac{\varphi}{k}} \quad (5)$$

The last equality in equation (5) arises from equation (3).

We have from equations (4), (5):

$$\frac{P_C(S_w)}{P_C(S_{wm})} = \frac{\bar{r}}{r_{threshold}(S_w)} \quad (6)$$

The quantity  $\left(\frac{\bar{r}}{r_{threshold}(S_w)}\right)$  is a function of solely, the wetting fluid saturation  $S_w$  and the pore structure of the rock, and is independent of  $\phi$  the value of the porosity of the rock. Let  $F(S_w)$  denote this function.

$$F(S_w) = \frac{\bar{r}}{r_{threshold}(S_w)} = \frac{P_C(S_w)}{P_C(S_{wm})} \quad (7)$$

$F(S_w)$  relates  $P_C(S_{wm})$  to  $P_C(S_w)$  through:

$$P_C(S_w) = P_C(S_{wm})F(S_w) \quad (8)$$

We have, substituting for  $P_C(S_{wm})$  from equation (5) into equation (8):

$$P_C(S_w) = \frac{2\lambda\sigma\cos\theta}{c} \sqrt{\frac{\phi}{k}} F(S_w) \quad (9)$$

Equation (9) rearranges to:

$$\frac{\lambda}{c} F(S_w) = \frac{P_C(S_w)}{2\sigma\cos\theta} \sqrt{\frac{k}{\phi}} \quad (10)$$

The function  $\frac{P_C(S_w)}{2\sigma\cos\theta} \sqrt{\frac{k}{\phi}}$  is called as the Leverett J-Function denoted as  $J(S_w)$  defined as:

$$J(S_w) = \frac{P_C(S_w(r))}{2\sigma\cos\theta} \sqrt{\frac{k}{\phi}} \quad (11)$$

It can be noted that the RHS of equation (10) above is nothing but  $J(S_w)$ . Thus,

$$J(S_w) = \frac{\lambda}{c} F(S_w) = \sqrt{8}\lambda\tau \frac{\bar{r}}{r_{threshold}(S_w)} \quad (12)$$

Equation (12) the physical meaning of the Leverett  $J$  Function can be understood that the function expresses how the radius of the largest undrained radius at a given wetting-fluid saturation, scales with reference to the mean pore radius associated with the pore space, as the wetting fluid saturation varies. It is a function which is a reflection of the pore structure of the rock, because, the quantities that define this function, as portrayed at the RHS of the last equality in equation (12), can vary between any two given rocks only when the pore-size distribution and the pore structure of the two rocks differ.  $r_{threshold}(S_w)$  is purely a reflection of the pore structure of the rock.  $\bar{r}$  reflects the pore volume distribution over the pore radius, for a rock,  $\tau$  the tortuosity associated with the pore space of a rock is an expression of the pore structure of the rock. The value of  $\lambda$  the representative pore size to pore throat size, can be equal between two rocks only when their pore structures are very similar. Thus, Leverett  $J$  function represents an attribute of pore space by which rocks of different bulk porosity and permeability can be compared. Rocks which have the same  $J$  function are essentially equivalent from the perspective of the pore space. This paper is concerned with quantitatively understanding this attribute through modelling this attribute using NMR log data which is essentially expressive of the pore size distribution and the pore size to pore throat ratio, when considered along with the bulk properties of the porosity and permeability of the pore space of a rock.

## 1.2 Modelling the Leverett J-Function using NMR log data

$$\text{We have, } \frac{1}{T_2} = \rho \frac{S}{V} + \frac{1}{T_{2B}} + \frac{D(\gamma G T_E)^2}{12} \cong \rho \frac{S}{V} \quad (13)$$

(Coates et al, 2000; Pape et al, 2006)

Here,  $\rho$  stands for the transverse relaxivity of the grain surface,  $S, V$  respectively stand for the cumulated wetted surface area of the grains and the total pore volume, of the rock.  $T_{2B}$  stands for the bulk transverse relaxation time of the pore fluid.  $D$  stands for the coefficient of diffusion of the pore fluid.  $\gamma$  stands for the gyromagnetic ratio of the proton.  $G, T_E$  respectively stand for the magnitude of the gradient of the static magnetic field and the value of the inter-echo spacing employed for the acquisition of NMR echoes.

The approximation indicated at equation (13) is valid for rocks which have negligible grain dissolution porosity and fractures, and where the NMR data acquisition parameters are the normal field gradient

and inter-echo spacing parameters employed. A further requirement, assumed to be satisfied by the rocks on which the application of the methodology of modelling the Leverett J-Function is discussed here, is that the pore fluid is liquid, so that the value of  $D$  is not too high. This ensures that inter-pore diffusive coupling of spins is low enough so as to not interfere with the rendering of the pore level surface area to volume ratio distribution of the rock, by the  $T_2$  distribution. Let  $\alpha$  denote the shape factor of the pores, which in the present context would have the value 3.0, assuming spherical / approximately spherical pore shape assumed in this paper employing the fractal model of the pore space (see below). We have:

$$\frac{\alpha}{r} = \left(\frac{S}{V}\right)_{pore} \quad (14)$$

Let  $T_2(r)$  denote the value of  $T_2$  associated with a pore class differentiated by the pore radius having magnitude around  $r$ . Equation (14) implies:

$$T_2(r) = \frac{r}{\alpha\rho}, r = \alpha\rho T_2 \quad (15)$$

The NMR  $T_2$  distribution is the distribution of the pore volume over  $T_2$ . This distribution, denoted by  $\Psi(T_2)$  is defined, where  $dV_p$  is pore volume hosted by pores of  $T_2$ -value within  $(T_2, T_2 + dT_2)$ , by

$$dV_p = \Psi(T_2)dT_2. \quad (16)$$

The mean value of  $T_2$  is defined as the pore volume weighted average of  $T_2$  and is denoted by the symbol  $T_{2mean}$ .  $T_{2mean}$  is defined by:

$$T_{2mean} = \frac{\int_{T_{2min}}^{T_{2max}} T_2 \Psi(T_2) dT_2}{\int_{T_{2min}}^{T_{2max}} \Psi(T_2) dT_2} \quad (17)$$

### 1.2.1 Modelling the Leverett J-Function with no prior model of pore size distribution assumed

Consider a stage of an intrusion experiment where the the maximum value of the radius of the drained pores at that stage of intrusion is  $r_{threshold}$ . This wetting phase saturation, denoted as  $S_w(r_{threshold})$  is:

$$S_w(r_{threshold}) = \frac{\int_{T_2=r_{min}/\alpha\rho}^{T_2=r_{threshold}/\alpha\rho} \Psi(T_2) dT_2}{\int_{T_2=r_{min}/\alpha\rho}^{T_2=r_{max}/\alpha\rho} \Psi(T_2) dT_2} \quad (18)$$

Using the notation  $\psi(T_2) = \int_{T_{2min}}^{T_2} \Psi(T_2) dT_2$ ,  $T_{2min}$ ,  $T_{2max}$  denoting the minimum and maximum limits of  $T_2$  in the distribution. Further,  $\bar{r}$  is the pore volume weighted average pore radius. We then have,

$$T_{2min} = \frac{r_{min}}{\alpha\rho} \quad (19)$$

$$T_{2max} = \frac{r_{max}}{\alpha\rho} \quad (20)$$

$$T_{2mean} = \frac{\bar{r}}{\alpha\rho} \quad (21)$$

Equation (18) can be stated, in light of equations (19), (20) as:

$$S_w(r_{threshold}) = \frac{\int_{T_{2min}}^{T_2(r)} \Psi(T_2) dT_2}{\int_{T_{2min}}^{T_{2max}} \Psi(T_2) dT_2} = \frac{\psi(T_2)}{\psi(T_{2max})} = \frac{\psi(r_{threshold}/\alpha\rho)}{\varphi_{NMR}} \quad (22)$$

The subscript 'NMR' in  $\varphi_{NMR}$  in equation (20) indicates that the quantity is the estimate of the porosity by the NMR data. The LHS of equation (22) is the wetting phase saturation as per NMR data. The last equality of equation (22), follows from the fact that  $\psi(T_{2max})$  is equal to  $\varphi_{NMR}$ .

Since the function  $\psi$  is readily evaluated from the NMR  $T_2$  distribution as seen from the above analysis, the RHS of equation (22) is readily evaluated for any given value of  $r_{threshold}$ . Thus, the function  $S_w(r_{threshold})$  is readily evaluated from NMR  $T_2$  distribution using equations (19) – (22). Note that  $\psi$  is a known function, at this point, and hence, the function  $\psi^{-1}$  is also determined at this point. Therefore, the function  $r_{threshold}(S_w)$  is also determined, through the inversion of equation (22) as,

$$r_{threshold}(S_w) = \alpha\rho\psi^{-1}(\varphi_{NMR}S_w) \quad (23)$$

Now, the value of  $T_{2mean}$  is readily evaluated from NMR  $T_2$  distribution using equation (17) and  $\bar{r}$  is therefore evaluated using equation (19) as:

$$\bar{r} = \alpha\rho T_{2mean} \quad (24)$$

Substituting for  $\bar{r}$ ,  $r(S_w)$  from equations (24), (23) into equation (12) and simplifying:

$$J(S_w) = J(S_w) = \frac{\lambda}{c} \frac{1}{\varphi_{NMR}} \frac{\alpha\rho T_{2mean}}{\psi^{-1}(\varphi_{NMR}S_w)} \quad (25)$$

Equation (25) is also stated as:

$$J(S_w) = \frac{\sqrt{8}\lambda\tau}{\varphi_{NMR}} \frac{\alpha\rho T_{2mean}}{\psi^{-1}(\varphi_{NMR}S_w)} \quad (26)$$

### 1.2.2 Modelling the Leverett J-Function assuming a fractal model of pore space

The pores are assumed to be of spherical shape or of a shape approximating to a spherical shape, and so, projecting as circles on a plane section. The number of circles on a plane section whose radius exceeds a given value  $r$  would scale with respect to the value of  $r$  chosen, as (Yu and Li, 2001; Yu and Cheng, 2002):

$$n(> r) = \zeta \left( \frac{r_{max}}{r} \right)^{D_f} \quad (27)$$

Here, the LHS stands for the number of circles per unit area of the plane section referred to above, whose radius exceeds  $r$ .  $\zeta$  is a constant and  $D_f$  is the fractal dimension of the distribution.

Let  $n(r)$  denote the number density distribution of the pores distributed over  $r$ . Let  $r_{min}$ ,  $r_{max}$  denote the minimum and maximum pore radius within the pore space respectively.

It is noted, using equation (27) that:

$$n(r)dr = n(> r) - n(> r + dr) = \chi D_f r^{-D_f-1} dr \text{ where } \chi = \zeta(r_{max})^{D_f} \quad (28)$$

It is a pre-requisite of fractal models supposed to be applicable to modelling pore space, that the maximum pore dimension is much larger than the minimum pore dimension. This stipulation, (which is concisely stated as the condition  $(r_{max}/r_{min}) \gg 1$ , is met with, for the case of most of the porous rocks.

The wetting fluid saturation at any stage of an intrusion experiment, when the threshold radius is  $r_{threshold}$  is given by:

$$S_w(r_{threshold}) = \frac{\int_{r_{min}}^{r_{threshold}} \frac{4}{3}\pi r^3 n(r) dr}{\int_{r_{min}}^{r_{max}} \frac{4}{3}\pi r^3 n(r) dr} \quad (29)$$

Substituting for  $n(r)dr$  from equation (28) in equation (29) and simplifying,

$$S_w(r_{threshold}) = \frac{\int_{r_{min}}^{r_{threshold}} r^{2-D_f} dr}{\int_{r_{min}}^{r_{max}} r^{2-D_f} dr} = \frac{r^{3-D_f} - r_{min}^{3-D_f}}{r_{max}^{3-D_f} - r_{min}^{3-D_f}} \quad (30)$$

Since,  $r_{max} \gg r_{min}$  equation (30) can be stated as:

$$S_w(r_{threshold}) = \left( \frac{r_{threshold}}{r_{max}} \right)^{3-D_f} \quad (31)$$

We have, from the definition of mean pore radius and the assumption of spherical to near spherical pore shape:

$$\bar{r} = \frac{\int_{r_{min}}^{r_{max}} \frac{4}{3}\pi r^3 n(r) dr}{\int_{r_{min}}^{r_{max}} \frac{4}{3}\pi r^3 n(r) dr} = \frac{\int_{r_{min}}^{r_{max}} r^4 \chi D_f r^{-D_f-1} dr}{\int_{r_{min}}^{r_{max}} \frac{4}{3}\pi r^3 \chi D_f r^{-D_f-1} dr} = \left( \frac{3-D_f}{4-D_f} \right) \left( \frac{r_{max}^{4-D_f} - r_{min}^{4-D_f}}{r_{max}^{3-D_f} - r_{min}^{3-D_f}} \right) \quad (32)$$

(Since, as per equation (28),  $n(r)dr = \chi D_f r^{-D_f-1}$ ). Further, since  $r_{max} \gg r_{min}$ , equation (32)

can be simplified to the approximate relation:

$$\bar{r} = \left(\frac{3-D_f}{4-D_f}\right) r_{max} \quad (33)$$

Equation (33) can be stated, using equations (29), (20) as:

$$\bar{r} = \alpha\rho \left(\frac{3-D_f}{4-D_f}\right) T_{2max} \quad (34)$$

Equation (32) implies,

$$r_{threshold}(S_w) = r_{max} S_w^{-(3-D_f)} = \alpha\rho T_{2max} S_w^{-(3-D_f)} \quad (35)$$

The last equality in equation (35) arises from equation (20).

Substituting for  $\bar{r}$  from equation (34) and for  $r(S_w)$  from equation (35) into equation (12) and simplifying, we get,

$$J(S_w) = \frac{\lambda}{c} \left(\frac{3-D_f}{4-D_f}\right) S_w^{(3-D_f)} = \sqrt{8}\tau\lambda \left(\frac{D_f}{D_f-1}\right) \left(\text{Since, } C = \frac{1}{\sqrt{8}} \frac{1}{\tau}\right) \quad (36)$$

The modelling of  $J(S_w)$  is still incomplete because, the value of  $D_f$  is unknown. A method of evaluating  $D_f$  is discussed now. Consider equation (21) and equation (31). It is assumed that the wetting fluid saturation computed from NMR data using equation (21) is a good estimate of the wetting fluid saturation, and so is the computed wetting fluid saturation computed as per equation (31) based on an analysis which assumes the fractal model of pore space. Therefore, the LHS of equations (22), (31) can be considered as representing the same saturation and hence that the LHS of the two equations can be equated. We then have,

$$\frac{\psi(r_{threshold}/\alpha\rho)}{\varphi_{NMR}} = \left(\frac{r_{threshold}}{r_{max}}\right)^{3-D_f} \quad (37a)$$

Equation (37a) can be stated for the generalized pore radius variable, as:

$$\frac{\psi(r/\alpha\rho)}{\varphi_{NMR}} = \left(\frac{r}{r_{max}}\right)^{3-D_f} \quad (37b)$$

Using equations (15), (19), equation (37b) is stated as:

$$\frac{\psi(T_2)}{\varphi_{NMR}} = \left(\frac{T_2}{T_{2max}}\right)^{3-D_f} \quad (38)$$

Taking logarithms on both sides of equation (38) and rearranging:

$$\log \psi(T_2) - \log \varphi_{NMR} = (3 - D_f) \log \left(\frac{T_2}{T_{2max}}\right) + \log \varphi \quad (39)$$

Equation (39) suggests that a plot of  $\log \psi(T_2)$  against  $\log \left(\frac{T_2}{T_{2max}}\right)$  should be a straight-line having slope  $(3 - D_f)$  and intercept  $\log \varphi_{NMR}$ . The value of the fractal dimension  $D_f$  is found by making the plot mentioned for different values of  $T_2$ . Once  $D_f$  is evaluated as above, all parameters in the RHS of equation (34) are evaluated and hence the  $J$  function is evaluated for any given value of the wetting fluid saturation  $S_w$ .

### 1.3 Modelling the Saturation – Height Function using NMR data

From equation (22),  $S_w(r(P_C)) = \frac{\psi(r/\alpha\rho)}{\varphi_{NMR}} = \frac{\xi(P_C)}{\varphi_{NMR}}$  (say)  $\left(r = \lambda \frac{2\sigma\cos\theta}{P_C}\right)$ . Since the form of the function  $\psi$  is known, the equation above, defines a saturation - height function.

From equation (31),  $S_w(r(P_C)) = S_w(P_C) = \left(\frac{P_B}{P_C}\right)^{3-D_f}$ . This equation defines a saturation – height function as  $P_B = \lambda \frac{2\sigma\cos\theta}{r_{max}}$  is known as  $r_{max}$  can be obtained from the NMR  $T_2$  distribution, and the value of  $D_f$  is known.



## 1.4 Discussion

The two approaches demonstrated for modelling Leverett J-Function respectively culminate in equation (26) and equation (36) respectively.  $\phi_{NMR}$ ,  $T_{2logmean}$  are standard outputs of NMR data post-processing. The best estimate of  $T_{2max}$  is the  $T_{2logmean}$  of the 95-percentile of the  $T_2$  distribution (Ramamoorthy et al 2006). The representative value of  $\lambda$  is obtained by comparing the  $T_2$  distribution with the pore throat size distribution computed from MICP data and computing the logarithmic shift required to be applied on the  $T_2$  distribution for it to match the MICP data-derived pore throat size distribution (Coates et al, 2006; Coates et al, 1991). The procedure indicated above is to be carried out on the data of equivalent formation against which both NMR data and the MICP data are available on core plugs. In case laboratory NMR data is not available the NMR log based  $T_2$  distribution can be used. The value of  $\rho$  is determined from laboratory NMR investigation on core plugs of the rock / rock type for which the Leverett J-Function is modelled. Dividing the value of  $\lambda\rho$  referred to above, by the value of  $\rho$  determined in the laboratory as referred to above, gives an estimate of the value of  $\lambda$  the pore size to pore throat size ratio applicable for the modelling of the Leverett J-Function. The value of  $\tau$  the tortuosity associated with the pore space of the rock can be obtained using the relation:

$$\tau^2 = F\phi \quad (43)$$

Here,  $F$  stands for the electrical formation factor of the rock. The quantity  $F\phi$  estimates the value of the square of the electrical tortuosity of the pore space. Equation (43) is based on the assumption that the value of the hydraulic tortuosity of the pore space of a rock is well estimated by the value of the electrical tortuosity of the pore space of the rock.

## 1.4 Conclusions

- Two methods of modelling the Leverett J-Function using NMR data are demonstrated
- Two methods of computing the Saturation - Height Function using NMR data are demonstrated
- A perspective of the Leverett J-Function in terms of gross attributes of the pore space has been explored.

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