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Estimation of saturation height function using linear regression method

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Abstract

Saturation-height function has a significant role in reserve estimation in heterogeneous carbonate reservoirs which have substantial transition zone present over FWL (free water level). The saturation-height function can be used to anticipate the water saturation in the reservoir above the free water level where extensive core samples are available to provide capillary pressure, porosity and permeability in the reservoir.

This paper generates a second degree equation to calculate initial water saturation by linear regression method using LINEST function in MS Excel. A set of core data from thirty eight cores from four oil wells in a carbonate reservoir in the Western Offshore was used to establish this equation. Capillary pressure, water saturation, porosity and permeability of each core data set were used as inputs in the algorithm. The results have been validated with log derived saturations which show a good match.

This equation aids in predicting the water saturation of the reservoir considering effects of reservoir structural relief. However this algorithm can only become a better alternative to log derived saturation provided that the core data used for an area or field is as representative as well logs.

Introduction

Saturation height in (m) is a function of capillary pressure Pc in (psi) and difference in densities between wetting (water) and non-wetting (oil) phases at that height above the free water level (FWL). The equation is given by

$$H = \frac{Pcres.144}{\left(\frac{\rho w}{Bw}\right) - \left(\frac{\rho o}{Bo}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Where Pcres is the capillary pressure in reservoir condition and is computed from the laboratory core data using Equation 2.

$$Pcres = Pclab.\left(\frac{бwo}{бwg}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Pclab is capillary pressure of the core in laboratory condition, the values of бwo (surface tension between water and oil layer) and бwg (surface tension between water and gas layer) were taken as 25 and 75 dynes respectively

$$Pcres = \left(\frac{1}{3}\right).(Pclab) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

In this study, ρw and ρo are density of water and oil respectively in gm/cm^3.

Bw and Bo are Water formation volume factor and oil formation volume factor respectively in rbbl/stbbl.

Core Data

Capillary pressure data measured for 38 core plugs from 4 wells were used in this study. Table 1 illustrates the porosity and permeability of these samples. The wells were selected on the basis of their location in the field spreading over north, south, east and west extremes. The core samples were drilled from whole cores between depths 1509 m to 1582 m. Owing to the carbonaceous nature of the reservoir, porosity and permeability of the samples varies over a wide range. Air-water capillary pressure characteristics using porous –plate capillary were used for saturation height calculation.

Saturation height calculation

In this method initially the capillary pressure v/s saturation from core data was plotted. The plot is shown in Fig. 1.

A range of 24 Pc value (Air-water in atm) ranging from .029 atm to 12.081 atm was selected from Fig. 1 and their corresponding height (m) was calculated using equation (1). The final expression of saturation height after changing from lab units to oilfield units is as follows:

$$H\ (m) = \frac{14.7\ .\ 144\ .\ Pclab}{3\ .\ 62.5\ .\ \left(\left(\frac{\rho w}{Bw}\right) - \left(\frac{\rho o}{Bo}\right)\right)} \text{……………………} \quad (4)$$

Regression

The regression technique uses a group of random variables, thought to be predicting Y, arriving at a mathematical relationship between them. This relationship is typically in the form of a straight line (linear regression) that best approximates all the individual data points.When the regression line is linear (y = ax + b) the regression coefficient is the constant (a) that represents the rate of change of one variable (y) as a function of changes in the other (x).

The LINEST function calculates the statistics for a line by using the "least squares" method to calculate a straight line that best fits the data, and then returns an array that describes the line.

Linear Regression Method

Various techniques are present to generate a saturation height function that relates capillary pressure to porosity and permeability. While Leverett's J-function [2] approach is a classic technique, capillary pressure-based method by Johnson [3] and Log based method by Skelt-Harrison [4] are common methods applied for saturation height estimation.

In this method water saturation is related to porosity and permeability through the following equation

$$Sw = A + (B\ .\ phi) + (C\ .(phi)^2) + \big(D\ .(log(k))\big) + (E\ .(log(\ k))^2) \text{…………………………………} \quad (5)$$

In order to calculate the regression coefficient A, B, C, D and E LINEST function was used in excel.

Syntax for LINEST function: LINEST(known_y's, [known_x's], [const], [stats]) "Ctrl" and "Shift" keys are pressed along with "Enter" key to obtain an array as an output.

Where known Y is Sw and known X is an array of phi, phi^2, Log K, $(\text{Log K})^2$ for each core. The input data is illustrated in Table 2. Constant and stats are taken as true , true to get an array of regression statistics. The first true indicates that we want the line in the form of (y = ax + b) and the second true specifies that we wish to list the error estimates. Hence with the above equation a 5X5 array was generated for each set of 24 Pc values which described the best fit line for our data.

For Pc=0.029 atm the Least squares result is printed as shown below:

0.307378	-0.76432	-69.2463	36.36622	95.17682
0.094048	0.220617	25.34317	10.42174	1.017829
0.432958	0.569245	#N/A	#N/A	#N/A
6.299185	33	#N/A	#N/A	#N/A
8.164754	10.69332	#N/A	#N/A	#N/A

The content of the array is labeled below to show the meaning of each cell.

	A		B	C	D	E
Constant	95.17682	Slope	36.36622	-69.2463	-0.76432	0.307378
(±)	1.017829	(±)	10.42174	25.34317	0.220617	0.094048
s(y)	#N/A	r^2	#N/A	#N/A	0.569245	0.432958
Degree of Freedom	#N/A	F	#N/A	#N/A	33	6.299185
residual ss	#N/A	regression ss	#N/A	#N/A	10.69332	8.164754

The first two rows gives the values of A, B, C, D and E.

A= 95.17682 ± 1.017829, B= 36.36622 ± 10.42174, C= -69.2463 ± 25.34317, D= -0.76432 ± 0.220617 and E= 0.307378 ± 0.094048

r^2 is a rough indicator of the goodness of fit. It is an expression obtained by dividing regression sum of squares by total sum of squares. Total sum of squares is the sum of the squared deviations of the original data from the mean. Regression sum of squares is the sum of squared deviations of the fit values from the mean and residual sum of squares is the sum of squared residuals. s(y) is the standard deviation of the y values. F-statistic is the ratio of the variance in the data explained by the linear model to the variance unexplained by the model and gives an even better statistical test of the goodness of fit. The variance of y values is obtained by dividing the residual ss and regression ss by the degree of freedom. However in this study only the first row of the array is used for determining saturation by equation (5). The results are given in Table 3.

Comparison between Regressed Saturation and Log derived Saturation

In order to validate the results, processed logs were used for the well C-9. First the set of 24 Pc was converted to height using equation (4). The height of transition zone above free water level was calculated using the values for a range of capillary pressure. The processed log was digitized and for each recorded depth, the regressed Sw was calculated by using equation (5). The flow chart in Fig. 5 demonstrates the steps involved in computing regressed Sw. The Log derived saturation and the regressed saturation was plotted against depth as shown in Fig. 2. While Fig.3 shows a good match like Fig.2 but Fig. 4 shows considerable deviation between calculated saturation and log derived saturation.

Conclusions

In the presented technique, saturation values are calculated as a function of porosity, permeability and height above Oil-Water-Contact. However, if there is a wide variation in porosity, this method may yield higher error values. However this method is empirical in nature but can be made into a convenient alternative of the log derived saturation owing to the practical nature of the algorithm. However, it entails abundantly available core data, representing the field accurately, in order to apply it to the Dynamic reservoir model.

References

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Table 1: Porosity and permeability of 38 core samples.

Sample no.	Depth (m)	Phi (%)	K (mD)
C-9 CC1	1558.21	19.11	2.75
C-9 CC2	1560.88	20.41	8.90
C-9 CC3	1563.90	24.17	8.45
C-9 CC4	1567.00	24.63	9.65
C-9 CC5	1569.21	25.95	10.00
C-9 CC6	1573.13	20.62	1.10
C-9 CC7	1580.16	17.87	2.10
C-9 CC8	1582.41	22.73	3.02
A-9 CC-1	1527.67	28.18	24.00
A-9 CC-2	1528.41	22.78	4.10
A-9 CC-3	1528.89	18.72	2.20
A-9 CC-4	1529.69	17.46	2.70
A-9 CC-5	1531.68	23.80	4.30
A-9 CC-6	1532.59	8.87	0.95
A-9 CC-7	1535.07	10.77	0.80
A-9 CC-8	1535.67	18.39	2.80
A-9 CC-9	1537.03	28.86	3.70
A-9 CC-10	1537.22	24.69	6.15
A-9 CC-11	1541.09	17.95	3.80
A-9 CC-12	1542.01	21.26	2.60
A-9 CC-13	1543.17	24.53	8.30
A-9 CC-14	1552.12	12.34	0.56
B-9 CC-1	1509.10	28.14	7.90
B-9 CC-2	1510.25	29.88	84.00
B-9 CC-3	1512.35	22.41	7.40
B-9 CC-4	1514.40	25.69	4.90
B-9 CC-5	1521.15	30.10	3.10
B-9 CC-6	1571.35	27.40	230.00
B-9 CC-7	1573.00	26.91	222.05
38-3 CC-1	1511.44	10.11	1.00
38-3 CC-2	1514.05	15.33	0.15
38-3 CC-3	1516.65	10.40	0.41
38-3 CC-4	1521.85	15.56	1.30
38-3 CC-5	1529.75	30.55	15.00
38-3 CC-6	1533.05	22.63	3.00
38-3 CC-7	1539.60	25.56	1.90
38-3 CC-8	1547.70	15.78	0.12
38-3 CC-9	1554.75	7.69	0.02

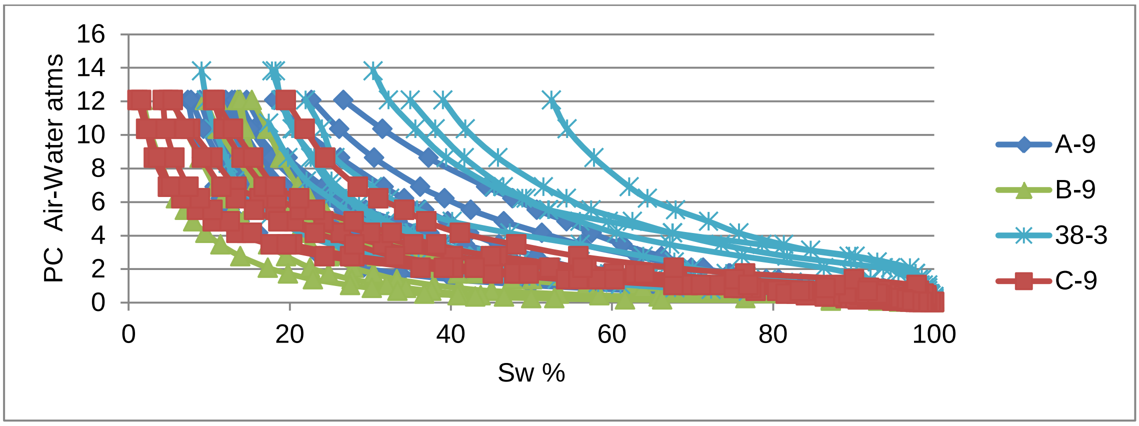
Table 2: Input table for LINEST function

Known Y	Known X's			
Sw	Phi	Phi^2	Log K	$(\text{Log K})^2$
100.00	0.19	0.04	0.44	0.19
100.00	0.20	0.04	0.95	0.90
100.00	0.24	0.06	0.93	0.86
99.43	0.25	0.06	0.98	0.97
97.70	0.26	0.07	1.00	1.00
99.59	0.21	0.04	0.04	0.00
98.51	0.18	0.03	0.32	0.10
98.09	0.23	0.05	0.48	0.23
99.84	0.28	0.08	1.38	1.90
99.55	0.23	0.05	0.61	0.38
99.40	0.19	0.04	0.34	0.12
99.38	0.17	0.03	0.43	0.19
99.44	0.24	0.06	0.63	0.40
98.04	0.09	0.01	-0.02	0.00
98.90	0.11	0.01	-0.10	0.01
99.30	0.18	0.03	0.45	0.20
99.73	0.29	0.08	0.57	0.32
99.77	0.25	0.06	0.79	0.62
99.46	0.18	0.03	0.58	0.34
99.40	0.21	0.05	0.41	0.17
99.67	0.25	0.06	0.92	0.84
98.69	0.12	0.02	-0.25	0.06
100.00	0.28	0.08	0.90	0.81
98.61	0.30	0.09	1.92	3.70
100.00	0.22	0.05	0.87	0.76
100.00	0.26	0.07	0.69	0.48
100.00	0.30	0.09	0.49	0.24
100.00	0.27	0.08	2.36	5.58
100.00	0.27	0.07	2.35	5.51
97.70	0.10	0.01	0.00	0.00
100.00	0.15	0.02	-0.82	0.68
98.03	0.10	0.01	-0.39	0.15
98.95	0.16	0.02	0.11	0.01
99.28	0.31	0.09	1.18	1.38
99.19	0.23	0.05	0.48	0.23
100.00	0.26	0.07	0.28	0.08
100.00	0.16	0.02	-0.92	0.85
100.00	0.08	0.01	-1.70	2.89

Table 3: Range of Pc values with corresponding regression coefficient.

Pc (atm)	H (m)	Height above OWC (m)	A	B	C	D	E
0.029	0.23587	1561.764	95.17682	36.36622	-69.2463	-0.76432	0.307378
0.063	0.512407	1561.488	94.86759	29.23306	-41.3377	-1.24124	0.387079
0.115	0.935345	1561.065	94.49717	20.13896	18.84018	-2.65462	-0.59111
0.184	1.496553	1560.503	99.40761	-50.1087	279.8153	-5.89141	-4.18998
0.270	2.196028	1559.804	99.15586	-58.4585	341.2452	-8.12186	-5.82655
0.356	2.895504	1559.104	97.68396	-50.8974	340.0664	-9.78206	-6.6336
0.443	3.603113	1558.397	94.35813	-22.5995	259.9816	-10.6242	-6.67376
0.529	4.302589	1557.697	92.72197	-14.8972	215.3362	-10.737	-6.8147
0.701	5.70154	1556.298	81.00395	93.80468	-114.378	-14.1951	-5.42663
0.874	7.108625	1554.891	75.33249	125.0485	-253.14	-15.7701	-4.03977
1.046	8.507577	1553.492	71.99589	126.3334	-312.715	-17.0735	-2.59988
1.391	11.31361	1550.686	71.99809	51.03724	-189.422	-18.8458	-0.52818
1.736	14.11965	1547.88	71.64215	-6.51955	-80.5489	-19.6337	0.759662
2.081	16.92569	1545.074	72.55005	-66.9789	37.0922	-20.1579	1.79699
2.770	22.52962	1539.47	73.15094	-153.611	217.8418	-19.0404	2.438722
3.460	28.1417	1533.858	70.41076	-199.576	325.8652	-17.3108	2.511216
4.150	33.75377	1528.246	67.79187	-231.017	407.507	-15.7268	2.479638
4.839	39.35771	1522.642	66.35664	-266.536	507.3095	-14.8201	2.49628
5.529	44.96978	1517.03	64.09144	-283.564	558.0685	-13.629	2.396409
6.219	50.58185	1511.418	61.06209	-281.655	559.6996	-13.1322	2.418614
6.908	56.18579	1505.814	58.11832	-279.491	564.6334	-12.8106	2.540502
8.632	70.20784	1491.792	50.59144	-253.129	534.7022	-12.4082	2.641191
10.356	84.22989	1477.77	44.50209	-227.953	497.6177	-12.2644	2.760488
12.081	98.26007	1463.74	39.11125	-202.736	460.4913	-12.3612	2.902973

Fig 1: Capillary pressure v/s saturation plot



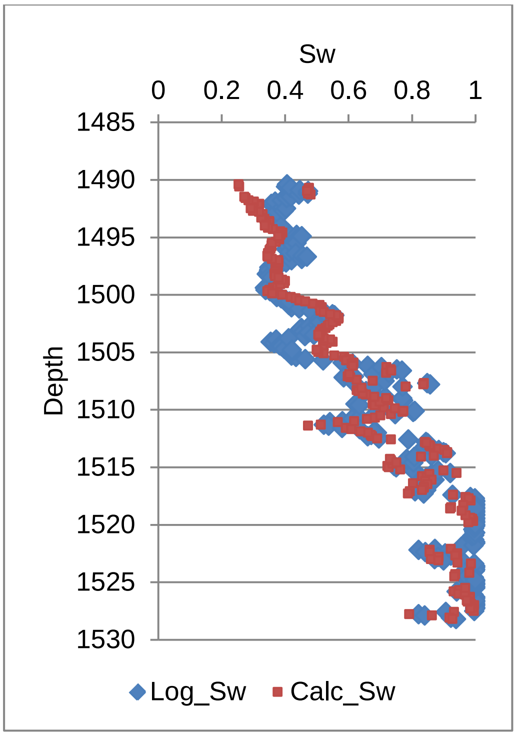


Fig 2: Comparison between Log derived Sw and Calculated Sw of well C-9

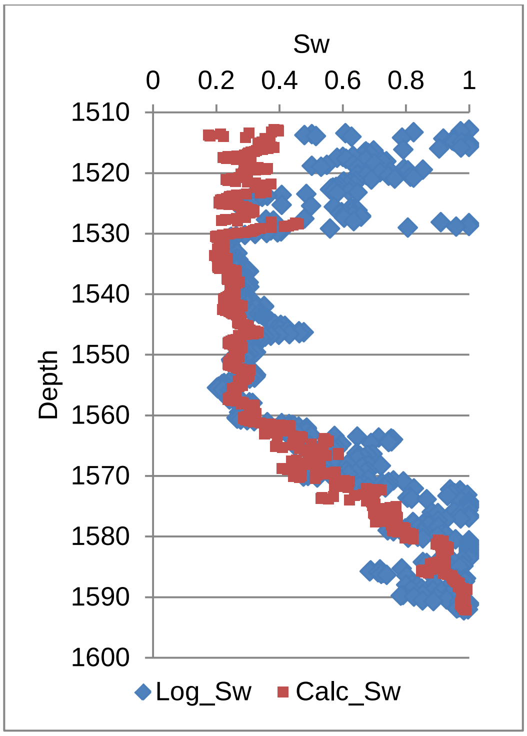


Fig 3: Comparison between Log derived Sw and Calculated Sw of well 15A-4

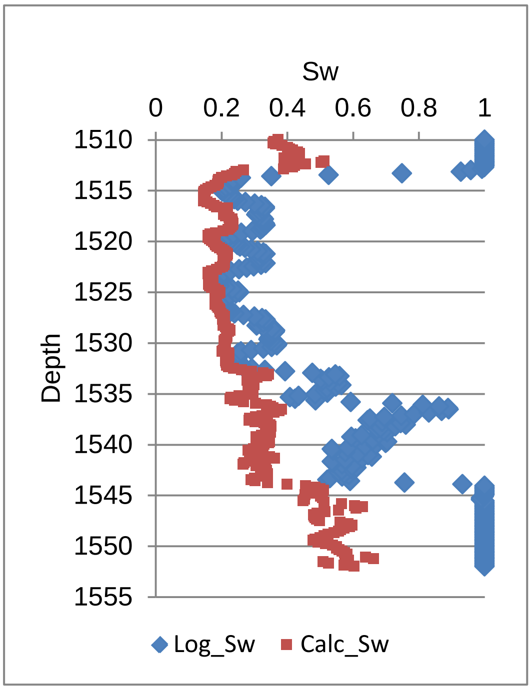


Fig 4: Comparison between Log derived Sw and Calculated Sw of well 38-2

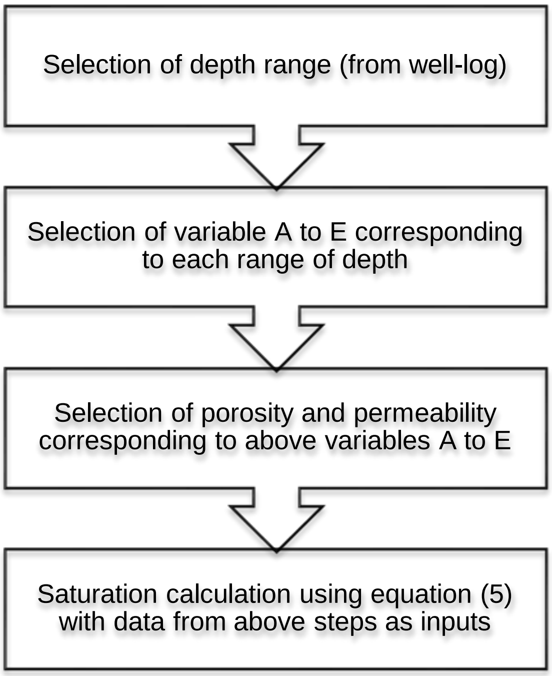


Fig 5: Flow chart to calculate Sw