Use of Pressure Transient Data Analysis for Multi-Porosity Reservoir Characterization

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Abstract

This paper presents reservoir heterogeneity characterization by fractional derivative approach to dynamic pressure transient data of oil/gas fields in multi-porosity naturally fractured reservoir with matrix participation in geological environment that are not possible by conventional techniques. The analysis of such type of data in reservoir engineering is known as "inverse problem" and has no unique solution. One can obtain information about transition zone responses; inter well and vertical permeability distribution in reservoir.

The goal of pressure transient data interpretation is to find out a reasonable estimates of reservoir properties such as anisotropy, fracture and matrix storativity, interporosity flow coefficient, existence of faults, fracture and matrix permeability and interference of two wells for better understanding of reservoir behavior. In reservoirs where majority of fluid flow is through fractures are sensitive to stress regime, & its perturbations. Production from reservoir or injection into the reservoir, cause changes in fluid pressure distribution in system. This will change the effective stress, which in turn will affect flow of hydrocarbons from reservoir through fracture network. The geoscientists such as geologists, geophysicists, petrophysicists, production and reservoir engineers have brought forward reasonable expectations by sharing their knowledge to integrate observed data in drilled wells to finding heterogeneity.

Automated computer matching techniques for observed pressured data and its calculated first and second derivative is used to estimate reservoir parameters to avoid human errors. This test data provide clues about the nature of fracture and multi-porosity network within reservoir and it help in extrapolating fracture from individual wells to between wells and ultimately in whole reservoir. Pressure first and second derivatives show different characteristics with different fracture patterns configurations. The estimated parameters from transient test data are used to generate 3-D model by combining other observed data from well-log and seismic by the use of geo-statistical methods.

Key Words: Pressure Transient; Reservoir Characterization; Fractional derivative.

Introduction

Most geological formations are fractured to some extent as a result of stress in the Earth's crust. Depending on the stress regime that a particular formation has been subjected to, these fractures many exist at many different length scales within the same formation. A naturally fractured reservoir is defined as a reservoir that contains a connected network of fractures created by natural processes. The most of the world's unconventional resources, such as shale gas, are also contained in fractured reservoirs. Earlier naturally fractured reservoir models are distinguished by the imposed model of matrix-fracture inter-porosity flow regime where all fractures are assumed to have identical properties. Fractures and faults affect all aspects of reservoir management from drilling and well placement production to stimulation, completion and EOR.

Analysis of the transient pressure response due to a change in the well rate serves as a powerful tool in evaluating and enhancing well performance. Equally important is the fact that transient test analyses aid in characterizing a reservoir. In some fields, reservoir performance is intensively monitored due to the existence of complex geology, an ongoing enhanced recovery operation, or monitoring of specific production issues. Emerging technologies employ digital sensors to continuously monitor bottom-hole pressure and flow rate during well production. Reservoir engineers place a high degree of confidence on well test interpretation in characterizing a reservoir and understanding well performance. In fact, well tests are regarded as reality checks in conceptualizing petroleum reservoirs.

During the last three decades, pressure transient in naturally fractured reservoirs has been extensively investigated. Many double porosity models were developed starting with the cube model, in which the matrix provides the storage while the fractures provide the flow medium. The double porosity models were first introduced by Barenblatt et al. [1] in a model that assumes pseudo steady state fluid transfer between the matrix and the fractures. Subsequently, Warren and Root [2] extended this model to a well test analysis and introduced it to the petroleum literature. The Warren and Root model was mainly developed for a transient well test analysis and introduced two parameters, ω and λ : ω describes the storativity of the fracture system, and λ is the parameter governing the fracture-matrix flow. The matrix contribution, then, reflects as a relatively steep linear transition zone. The transition zone behavior should be viewed as a means of deciphering matrix characteristics. Braester studied the effect of matrix block size on the transition curve. The dual porosity models fall into two categories based on the interporosity fluid transfer assumption (i) pseudo steady state and (ii) unsteady state models.

The dual porosity models assume uniform matrix and fracture properties throughout the reservoir, which may not be true in actual reservoirs. An improvement is to consider two matrix systems with different properties; this is a triple porosity system. A triple porosity model was proposed very recently. This model assumes the existence of two domains in the matrix and one domain in the fractures. Another form of triple porosity is to consider two fracture systems with different properties in addition to the matrix, sometimes referred to as a dual fracture model [3]. Two geometrical configurations were considered: A strata model and a uniformly distributed block model. In both the cases, two matrix systems have different properties flowing to a single fracture under gradient inter-porosity flow [4].

The transition zone behavior of the radial triple porosity system was then investigated. The strata model was extended by allowing the matrix systems to have different properties and thickness.

A radial triple-continuum model was presented in 2003. The system consists of fractures, a matrix system and a cavity system. Only the fractures feed the well, but they receive flow from both matrix and cavity system under pseudo steady state conditions.

As described by Van Everdingen and Hurst, the classical wellbore storage model is derived by considering of the conservation of wellbore mass and implicitly ignoring the momentum effects. There is also an implicit assumption that wellbore mass is a function of wellbore pressure only, since no other variables are present in the equation. Phase redistribution occurs in a well, which is shut in with gas and liquid flowing simultaneously. When such a well is shut in at surface, gravity effects cause the liquid to fall and the gas rise to the surface. The solution for such phenomena was presented by Fair [5] and Hegeman et al. [6] and Fair presented an exponential increase in storage, which is used to model wellbore redistribution. The Hegeman et al. [6] have assumed the exponential error form of change of wellbore storage effect. When this phenomenon occurs in a pressure buildup test the increased pressure in the wellbore are relieved through the formation and equilibrium will be attained eventually.

In the present paper we describe the reservoir heterogeneity characterization with the use of fractional derivative technique of pressure transient data with phase redistribution for multi porosity naturally fractured reservoir.

Definition of a Fractional Calculus and its Derivative

The fractional calculus is the branch of mathematics that deals with the generalization of derivatives and integrals to all real orders with various functions. We begin with the definition of a fractional derivative and its application since they are used in the formulation of the problems. Several definitions of a fractional derivative of a derivative and integral have been proposed. The most frequently used definition of a fractional derivative of order $\alpha > 0$ is the Riemann-Liouville definition, which is straightforward generalization to non-integer values of Cauchy formula. The Riemann-Liouville fractional derivative, which is defined as [7]

$$D^{\alpha}f(t) = \begin{cases} \frac{d^{n}}{dt^{n}} \left[\frac{1}{\Gamma(n-\alpha)} \int_{0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \right], n-1 < \alpha < n \\ \frac{d^{n}}{dt^{n}} f(t), \alpha = n \end{cases}$$

$$(1)$$

Where, \lceil is the Gamma function, m is positive integer such that n-1< α < n and α > 0.

An alternative definition of the fractional derivative was proposed by Caputo [7]

$$D^{\alpha}f(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_{0}^{t} (t-\tau)^{n-\alpha-1} \frac{d^{n}f(\tau)}{d\tau^{n}} d\tau, n-1 < \alpha < n \\ \frac{d^{n}}{dt^{n}} f(t), \alpha = n \end{cases}$$

$$(2)$$

The major utility of this type fractional derivative is caused by the treatment of differential equations of fractional order for physical applications, where the initial conditions are usually expressed in terms of a given function and its derivatives of integer order, even if the governing equation is of fractional order. Once these definitions are given, it is natural to write differential equations in terms of such quantities [8].

Mathematical Equations and their Solution

The pressure equation for the flow in fractured reservoir is given by

$$\frac{\partial^2 \mathbf{P}_{\mathrm{Df}}}{\partial \mathbf{r}_{\mathrm{D}}^2} + \frac{1}{\mathbf{r}_{\mathrm{D}}} \cdot \frac{\partial \mathbf{P}_{\mathrm{Df}}}{\partial \mathbf{r}_{\mathrm{D}}} = \omega_{\mathrm{f}} \cdot \frac{\partial \mathbf{P}_{\mathrm{Df}}^{\gamma}}{\partial \mathbf{t}_{\mathrm{D}}^{\gamma}} + \mathbf{v}_{\mathrm{m1}} + \mathbf{v}_{\mathrm{m2}}$$
(3)
where $\mathbf{r}_{\mathrm{c}} = \mathbf{r}_{\mathrm{c}} / \mathbf{r}_{\mathrm{c}}$ is $k_{\mathrm{m1}} \mathbf{r}_{\mathrm{w}}^2$ is $k_{\mathrm{m2}} \mathbf{r}_{\mathrm{w}}^2$

where,
$$r_{\rm D} = 1/|r_{\rm w}|, \lambda_1 = \frac{m}{k_{\rm f} \{(2-n)/2)h\}^2}, \lambda_2 = \frac{m}{k_{\rm f} \{(2-n)/2)h\}^2}, P_{\rm Df} = \frac{1}{q\mu B_0}, 0.4741,$$

 $t_{\rm D} = \frac{k_{\rm f} t}{[(n/2)\phi_{\rm m1}C_{\rm m1} + ((2-n)/2)\phi_{\rm m2}C_{\rm m2} + \phi_{\rm f}C_{\rm f})\mu r_{\rm w}^2]}; \omega_1 = \frac{(n/2)\phi_{\rm m1}C_{\rm m1}}{[(n/2)\phi_{\rm m1}C_{\rm m1} + ((2-n)/2)\phi_{\rm m2}C_{\rm m2} + \phi_{\rm f}C_{\rm f}]};$
 $\omega_2 = \frac{((2-n)/2)\phi_{\rm m2}C_{\rm m2}}{[(n/2)\phi_{\rm m1}C_{\rm m1} + ((2-n)/2)\phi_{\rm m2}C_{\rm m2} + \phi_{\rm f}C_{\rm f}]};$

In case of pseudo-steady state flow from matrix: $v_{m1} = \frac{n\lambda_1}{2}(P_{Df} - P_{Dm1}) \& v_{m2} = \frac{(2-n)\lambda_2}{2}(P_{Df} - P_{Dm2})$ (4)

For transient state matrix flow the value of vm1 and v_{m2} are as follows:

$$\mathbf{v}_{m1} = -(\mathbf{n}/2)^2 \lambda_1 \frac{\partial \Delta \mathbf{P}_{Dm1}}{\partial z_D} \bigg|_{z_d=0} \quad ; \quad \mathbf{v}_{m2} = ((2-\mathbf{n})/2)^2 \lambda_2 \frac{\partial \Delta \mathbf{P}_{Dm2}}{\partial z_D} \bigg|_{z_D=0}$$
(5)

The one layer behaves as pseudo-steady state and other layer as unsteady state then, $v_{m1} \& v_{m2}$ are as:

$$\mathbf{v}_{m1} = \frac{n\lambda_1}{2} (\mathbf{P}_{Df} - \mathbf{P}_{Dm1}); \quad \mathbf{v}_{m2} = ((2 - n)/2)^2 \lambda_2 \frac{\partial \Delta \mathbf{P}_{Dm2}}{\partial z_D} \bigg|_{z_D = 0}$$
(6)

The initial and boundary conditions are as follows

$$P_{Df} = P_{Dm1} = P_{Dm2} = 0, \text{ at } t_D = 0$$
 (7)

$$\lim_{\mathbf{r}_{\mathrm{D}}\to\infty}\mathbf{P}_{\mathrm{Df}} = 0; \left.\frac{\partial \mathbf{P}_{\mathrm{Df}}}{\partial \mathbf{r}_{\mathrm{D}}}\right|_{\mathbf{r}_{\mathrm{D}}=1} = -1; \mathbf{C}_{\mathrm{D}}(\partial \mathbf{P}_{\mathrm{wD}} / \partial \mathbf{t}_{\mathrm{D}} - (\partial \mathbf{P}_{\mathrm{Df}} / \partial \mathbf{r}_{\mathrm{D}})\right|_{\mathbf{r}_{\mathrm{D}}=1} = 1; \mathbf{P}_{\mathrm{wD}} = [\mathbf{P}_{\mathrm{Df}} - \mathbf{S}_{\mathrm{D}}(\partial \mathbf{P}_{\mathrm{Df}} / \partial \mathbf{r}_{\mathrm{D}}]_{\mathbf{r}_{\mathrm{D}}=1}$$
(8)

The Fair [5] has modified the van Everdingen and Hurst equation of sand face rate for constant wellbore storage effect by adding a term to account the pressure change due to phase redistribution as

$$(q_{\rm sf}/q) = q_{\rm wD} = 1 - C_{\rm D} [(dP_{\rm wD}/dt_{\rm D}) - (dP_{\phi \rm D}/dt_{\rm D}) - C_{\rm mD} \frac{d^2 q_{\rm wD}}{dt_{\rm D}^2}]$$
(9)

Thus, phase redistribution was modeled as a changing wellbore storage phenomenon. The pressure $P_{\phi D}$ has the following properties

$$\lim_{t \to 0} P_{\phi D} = 0, \quad (10a), \lim_{t \to \infty} P_{\phi D} = C_{\phi D}, \text{ a constant (10b)}, \quad \lim_{t \to \infty} (dP_{\phi D} / dt_D) = 0, \quad (10c)$$

The use of increasing/decreasing wellbore storage model to field data was first applied by Fair [3] as exponential form for changing storage pressure as

$$\mathbf{P}_{\boldsymbol{\varphi}\mathbf{D}} = \mathbf{C}_{\boldsymbol{\varphi}\mathbf{D}} \left(\mathbf{1} - \mathbf{e}^{-\mathbf{t}_{\mathrm{d}}/\boldsymbol{\alpha}_{\mathrm{D}}} \right), \tag{10d}$$

and the changing storage pressure function as Hegeman et al. [6] have assumed, $P_{\phi D} = C_{\phi D} erf(t_D / \alpha_D)$, where, $C_{\phi D}$ and α_D are changing storage pressure and time parameters.

The general solution for dimensionless wellbore pressure ($\overline{P}_{wD}(s)$) in terms of $\overline{P}_{fD}(s)$ and $\overline{P}_{\phi D}$ in Laplace space is obtained as

$$\overline{P}_{wD}(z) = \frac{\left[s\overline{P}_{fD} + S_{D}\right]\left[1 + C_{D}s^{2}\overline{P}_{\phi D}\right]}{s\left[1 + C_{mD}s^{2} + C_{D}s\{s\overline{P}_{fD} + S_{D}\}\right]}$$
(11)

$$\text{Where, } \overline{P}_{\text{fD}} = \frac{K_0(x)}{s[xK_1(x)]}, \text{ } x=\sqrt{\{\text{sf(s)}\}; } \overline{P}_{\phi \text{D}} = (C_{\phi \text{D}} / s) - C_{\phi \text{D}} / (s + \alpha_{\text{D}}^{-1}); \\ \overline{P}_{\phi \text{D}} = s^{-1}C_{\phi \text{D}}e^{\alpha_{\text{D}}^{2s^2/4}} erfc(\alpha_{\text{D}}s/2) \text{ and } s < \frac{1}{2} e^{-\frac{1}{2}(s^2/4)} e^{-\frac{1$$

$$\begin{split} f(s) = & [\omega_f + (\sqrt{n\omega_1\lambda_1} / \sqrt{2s}) \tanh\{\sqrt{2\omega_1s} / \sqrt{n\lambda_1}\} + \sqrt{(2 - n)\omega_2\lambda_2} / \sqrt{2s} \tanh\{\sqrt{2\omega_2s} / \sqrt{(2 - n)\lambda_2}\}], & \text{for both} \\ \text{matrix} & \text{behaves} & \text{as} & \text{transient} & \text{flow} & \text{and} \\ f(s) = & [\omega_f + n\omega_1\lambda_1 / (n\lambda_1 + 2\omega_1s) + \sqrt{(2 - n)\omega_2\lambda_2} / \sqrt{2s} \tanh\{\sqrt{2\omega_2s} / \sqrt{(2 - n)\lambda_2}\}], & \text{for one matrix behave} \\ \text{as pseudo-steady & other behave as transient steady state flow towards fracture and then to wellbore.} \end{split}$$

Conclusions

This paper describes the transient pressure response of multi porosity naturally fractured reservoir with changing wellbore storage effect by fractional derivative approach in heterogeneous reservoir. The characteristics of flow from both the matrix to fracture are of transient, pseudo-steady state and pseudo-steady state transient flow types. In reservoir where fractures are much more permeable than matrix, the derivatives of a pressure transient well test are a good indication of the underlying fracture network. The multi-porosity reservoir consists of two matrices and fractures with different properties have been considered for both matrix and fractures in present study. The fig.1 shows the dimensionless plot for both the matrix behaving as pseudo-steady state and fig. 2 shows the plot for one matrix as pseudo-steady state case of model of [8] and fig.4 is for model [3] where both matrices behaves as transient behavior of [3]. The various reservoir parameters including shape of matrix blocks are estimated by using the dynamic pressure transient test data. The effect of momentum on the wellbore pressure response has also been considered. It is noted that fracture permeability in a fractured reservoir is very sensitive to stress change due to pressure depletion or injection. We have identified different flow regimes in figs. 5 & 6 for horizontal well with pressure & its derivative data.

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References

- 1. G.I. Barenblatt, Iu.P. Zheltov, and I.N. Kochina, "Basic concept in the theory of seepage of homogeneous liquids in fissured rocks", J. Appl. Math. Mech.24, 5,1286-1303, 1960.
- 2. J.E. Warren, and P.J. Root, "The behavior of naturally fractured reservoirs", SPEJ, 245-55, Sept. 1963.

3. Y. Jalali and I. Ershaghi, "Pressure transient analysis of heterogeneous naturally fractured reservoirs", SPE paper 16341, 1987.

4. D. Bourdet, "Well Test Analysis: The Use of Advanced Interpretation Models", Elsevier Inc., Amsterdam, 2002.

5. W. B., Fair Jr., "Generalization of wellbore effects in pressure transient analysis", SPE paper 24715, 1992.

6. P. S., Hegeman, D. L. Hallford and J. A. Joseph, "Well test analysis with changing wellbore storage", SPE paper 21829, 1991.

7. I. Podlubny, Fractional Differential Equations, Academic Press, San Diego, 1999.

8. J. Dreier, "Pressure Transient analysis of wells in reservoirs with a multiple fracture network", M.S. Thesis, Colorado School of mines, Golden, Colorado, 2004.

9. Hasan A. Al-Ahmadi and R. A. Wattenbarger, "Triple porosity models: one further step towards capturing fractured reservoir heterogeneity", Saudi Aramco J. of Technology, 52-65, 2011.



Fig. 1 ω_1 =0.83, λ_1 =1.e-6, ω_2 =0.15, λ_2 =1.e-5, γ =0.98, n=0.3, S_D=0.5, C_D=10², C_{ω D}=1, α_D =25, C_{mD}=1.e+5



Fig. 3 ω_1 =0.83, λ_1 =1.e-7, ω_2 =0.15, λ_2 =1.e-6, γ =0.98, n=1.0, S_D=0.5, C_D=10^2, C $_{\omega D}$ =1, α_D =25, C $_{m D}$ =1.e+5





 $\begin{array}{l} \mbox{Fig. 2 } \omega_1 {=} 0.83, \, \lambda_1 {=} 1.e{-}6, \, \omega_2 {=} 0.15, \, \, \lambda_2 {=} 1.e{-}5, \, \gamma {=} 1, \, n {=} 0.3, \\ \mbox{S}_D {=} 0.5, \, \mbox{C}_D {=} 10^2, \, \mbox{C}_{\phi D} {=} 1, \! \alpha_D {=} 25, \, \mbox{C}_{m D} {=} 1.e{+}5 \end{array}$



Fig. 4 ω_1 =0.83, λ_1 =1.e-6, ω_2 =0.15, λ_2 =1.e-5, γ =0.98, n=0.3, S_D=0.5, C_D=10^2, C $_{eD}$ =0, α_D =0, C $_{mD}$ =0.



Fig. 6. Log-log plot of Δt vs. ΔP for horizontal well & identified all the flow regimes that exists.