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Design Analysis of the Raymer–Hunt-Gardner Algorithm and Modification for Shaly Sands

Abstract

Time average equations of MRJ Wyllie and Raymer et al have been under criticism for their inefficiency in dealing with the shaly sand acoustic porosity. As such the prevailing practice is to create regression algorithms that correlate acoustic velocity to porosity. Present work presents a modification of the Raymer-Hunt-Gardner algorithm to make it a universal velocity-porosity transform applicable to shaly sands.

Design analysis of the Raymer-Hunt-Gardner algorithm has revealed that porosity quadratic is implicit in it with scope for generalization to cover the shaly sand p-wave phenomena. Accordingly RHG relation has been generalized as $\phi^2 + \phi\left(\frac{\Delta t_{ma}}{\Delta t_f} - \omega\right) + (1 - \frac{\Delta t_{ma}}{\Delta t_{log}}) = 0$ to serve as a porosity algorithm to the shaly sand P-wave velocities. Porosity first order coefficient is modified by replacing 2 with ω, defined as a variable that addresses the clay content of the rock. Modified algorithm has been applied to a large data set of sandstone formations and ω is found to be in the range 1.0 to 10 or even greater subject to the degree of acoustic anomaly presented by the samples. For reasonably good porosity and low clay content ω (Omega) is found to be in the range 1.7 to 3 for sandstone velocity data of Han at 40 MPa pressure. Application of the generalized algorithm to to account for the scatter of the Δt-ϕ relation has shown that ω is an efficient variable to keep the residual a minimum by playing the role of a surrogate parameter that accounts for all the factors that have escaped modelization.

Further, the Modified Raymer-Hunt-Gardner algorithm as above has been shown to be of universal application by defining the variable omega in petrophysical terms as:

$$\phi^2 + \phi\left(\frac{\Delta t_{ma}}{\Delta t_f} - \left(\frac{\Delta t_s}{\Delta t_p}\right)^x\right) + (1 - \frac{\Delta t_{ma}}{\Delta t_{log}}) = 0$$

Algorithm as above can be tailor made to meet the specific shaly sand environment by fixing the V_p/V_s exponent x appropriately based on shale indicator logs or laboratory studies. It is shown that in terms of shale content omega may be defined as $\omega = \frac{V_p}{V_s} + \rho_b C_{frac} = \left(\frac{V_p}{V_s}\right)^x$

For Han's data, the Omega may be comfortably derived as $(V_p/V_s) + \rho_b * C_{frac}$ where C_{frac} is the volume fraction of clay/shale. It has been shown that in combination with the density log and a

clay indicator such as GR, P-wave velocity can be used for deriving precise porosity values. Experience also suggests bulk density may be substituted for omega to obtain porosity values comparable to density-neutron porosity in shaly sands. Modified equation can be applied to derive porosity from S-wave velocity also using Δt_{ma} (shear) and retaining the p-wave velocity of the fluid. It is likely that omega may emerge as a parameter capable of characterizing the shaly sands in a meaningful manner. Value addition from acoustic logs and especially, Dipole Sonic Shear Imager (DSI) log will be greatly improved by the new method suggested.

I. Introduction

The Wyllie (1956)[1] time average equation is widely used to predict porosity from sonic compressional wave velocity in petrophysical applications such as sonic log interpretation[2,3]. But despite its wide prevalence, hunt had been there for improved algorithms that gives a better description of the correlation between P-wave velocity of the water saturated rock ($V_p = V_{log}$) and porosity (ϕ). Wyllie equation expresses interval transit time as a volumetric weighted average of the interval travel times of the fluid (Δt_f) and the matrix (Δt_{ma}) and is represented differently in many forms like:

$$\Delta t_{log} = \phi \Delta t_f + (1-\phi)\Delta t_{ma} \qquad \text{-------------------------------------} \qquad (1a)$$

Re-written in the slope intercept form:

$$\frac{1}{V_{log}} = \phi\left[\frac{1}{V_f} - \frac{1}{V_{ma}}\right] + \frac{1}{V_{ma}} \qquad \text{-------------------------------------} \qquad (1b)$$

The simple time average relation as above is assumes the sedimentary rocks to be of uniform mineralogy and fluid saturated with high effective pressure. Proportionate addition of the transit times in solid and fluid phases is justified only when the wavelength is small in comparison to pore size and grain size. Further, it is assumed that the pores and grains make up homogeneous layers perpendicular to the acoustic wave and for best fit with the observed data, the rock has to be at high enough effective stress such as 30 MPa at which the terminal velocity is reached.

It is apparent from the relationship that the slope of the linear V^{-1}-ϕ trend is decided by the pore-fluid velocity vis-à-vis contrast with the matrix velocity. Expression as above could achieve only limited success in acoustic long interpretation as gas or low Vp fluid and the clay effect defied all modelization attempts[4]. Further, among all pore parameters (volume, shape and size) only pore volume could be incorporated by Wyllie et al. Yale's review (1985) of the literature had brought out wide discrepancies between the measured and predicted values of porosity.

Raymer et al (1980)[5] brought out the empirical algorithm as an alternative to the time average equation for acoustic log interpretation. This has been found to be variously expressed for different range of porosities and its general non-linear form is:

$$V_p = (1-\phi)^2 V_{ma} + \phi V_f \qquad \text{---------------} \qquad (2a)$$

In terms of sonic travel time -

$$\frac{1}{\Delta_{tlog}} = \left\{\left[\frac{(1-\phi)^2}{\Delta_{tma}}\right] + \left[\frac{\phi}{\Delta_{tf}}\right]\right\} \qquad \text{-------------} \qquad (2b)$$

V_p is the P wave velocity usually referred as the acoustic log of the formation and V_{ma} and V_f are the acoustic P wave velocities of the rock matrix and the fluid respectively. This new empirical transform was based on extensive field observations of transit time versus porosity data and suggested more consistent matrix velocities for given rock lithology over the entire porosity without the need to determine any compaction factor or other correction factors.

Han's work[6] attempted to correlate the velocities with the clay content as well and produced the empirical relationships like

$$V_p = 5.59 - 6.93\phi - 2.13C \; or = (5.59 - 2.13C) - 6.93\phi \qquad \text{--------------} \qquad (3a)$$

and

$$V_s = 3.52 - 4.91\phi - 1.89C \; or = (3.52 - 1.89C) - 4.91\phi \qquad \text{----------------} \qquad (3b)$$

Where velocities are in km/sec and C is the clay fraction.

II. Introducing the Acoustic Anomaly Factor

Recourse to regression equations for velocity porosity transforms is illustrative that the existing algorithms such as Wyllie and Raymer-Hunt-Gardner (RHG) fail in handling the spectrum of acoustic responses seen in sedimentary rocks. This difficulty can be overcome by modifying the RHG equation by incorporating what may be described as an "acoustic anomaly" factor ω as indicated below:

Design analysis of the original Raymer-Hunt-Gardner non-linear relationship leads to a quadratic in porosity to represent the p-wave phenomenon of sedimentary rocks.

$$\phi^2 + \phi\left(\frac{\Delta t_{ma}}{\Delta t_f} - 2\right) + \left(1 - \frac{\Delta t_{ma}}{\Delta t_{log}}\right) = 0 \qquad \text{----------} \qquad (4a)$$

This may be generalized to meet any global sedimentary environment such as shaly sands by modifying the first order coefficient as shown in 5(a) below:

$$\phi^2 + \phi\left(\frac{\Delta t_{ma}}{\Delta t_f} - \omega\right) + \left(1 - \frac{\Delta t_{ma}}{\Delta t_{log}}\right) = 0 \qquad \text{-----------} \qquad (4b)$$

-where the variable ω *(omega)* may be fixed on the basis of core data for minimum scatter and the impact of ω may be understood from a spreadsheet exercise.

Putting a =1, b = $\left(\frac{\Delta t_{ma}}{\Delta t_f} - \omega\right)$, c = $(1 - \frac{\Delta t_{ma}}{\Delta t_{log}})$, we may express the solution for porosity as:

$$\phi = -\frac{b}{2a} - \sqrt{\left(\frac{b}{2a}\right)^2 - \left(\frac{c}{a}\right)} = -\frac{b}{2a}\left[1 - \sqrt{1 - \frac{\left(\frac{c}{a}\right)}{\left(\frac{b}{2a}\right)^2}}\right] = -\frac{b}{2}\left[1 - \sqrt{1 - \frac{c}{\left(\frac{b}{2}\right)^2}}\right]$$

It is apparent that the output of the equation depends on the contrast of *c* and *b* and the slope of the quadratic in ϕ is obtained as –

$$2\phi + \left(\frac{\Delta t_{ma}}{\Delta t_f} - \omega\right) \qquad \text{-------------------} \qquad (5)$$

It is apparent that the slope of the algorithm is a function of the coefficient *b* i.e. slope = $2\phi + b$ where b = $\left(\frac{\Delta t_{ma}}{\Delta t_f} - \omega\right)$, is decided by the variable ω introduced to represent the scatter in the output of the Raymer-Hunt-Gardner algorithm. The three terms of the modified quadratic addresses different dimensions of the sediments:

Term-1:

ϕ^2 = Independent porosity contribution

Term-2:

$\phi\left(\frac{\Delta t_{ma}}{\Delta t_f} - \omega\right)$, sum of matrix, fluid and clay effect modified by porosity

Term-3:

$(1 - \frac{\Delta t_{ma}}{\Delta t_{log}})$, matrix travel time and the log value. When $\phi \rightarrow 0$, $\Delta t_{log} \rightarrow \Delta t_{ma}$

Equation as above can be applied to Han's data of P and S-wave velocities for deriving the best fit ω values that corresponds to zero difference for porosity derived from V_p and V_s. For P and S we may denote ω respectively as ω_p and ω_s[7].

III. Physical Expression for Acoustic Anomaly Factor ω

In the applications of the above modified algorithm with arbitrary values between 1.5 to 4 the surrogate variable ω has shown significant variation with respect to porosity and clay content and thus necessitates determination through core studies for a particular environment. Choice of general/default values of say ω_p=2 or ω_s = 3 is found to cause departure to the tune of 2-3 p.u even in the medium porosity range. Computational scenario can be much improved by defining ω in terms of V_p and V_s as shown below:

$$\phi^2 + \phi\left(\frac{\Delta t_{ma}}{\Delta t_f} - \left(\frac{V_p}{V_s}\right)^x\right) + \left(1 - \frac{\Delta t_{ma}}{\Delta t_{log}}\right) = 0 \quad \text{--------} \qquad (6a)$$

or

$$\phi^2 + \phi\left(\frac{\Delta t_{ma}}{\Delta t_f} - \left(\frac{\Delta t_s}{\Delta t_p}\right)^x\right) + \left(1 - \frac{\Delta t_{ma}}{\Delta t_{log}}\right) = 0 \quad \text{--------} \qquad (6b)$$

In the above, x is a variable that transforms (V_p/V_s) to ω. If we use the dry rock ratio (V_p/V_s) =1.6, it may prove insufficient to give the correct porosity for clean sands. In terms of shaliness or clay content (C_{frac}), $\omega_p = (V_p/V_s)^x$ may be defined as –

(a) ω as a function of V_p/V_s , bulk density and clay content

$$\omega_p = \frac{V_p}{V_s} + \rho_b C_{frac}$$

(b) ω as a function of bulk density and P-wave velocity

$$\omega_p = \frac{(2\rho_b + V_p/V_s)}{3}$$

Application to Han's Data

Both the above propositions (a) and (b) can be tested on Han's data and Table-1 below presents the data:

Sample No	ρ	Clay Fraction	V_p	V_s	Porosity	ω_p for 0 Diff: in ϕ	Cas (a)		Case (b)	
							ω_p	ϕ diff:	ω_p	ϕ diff:
1	2	3	4	5	6	7	8	9	10	11
56	2.12	0.12	3.17	1.77	0.2945	2.04	2.05	0.002	2.01	-0.007
55	2.09	0.11	3.2	1.75	0.2993	2.00	2.06	0.014	2.00	0.000
7	2.24	0.16	3.36	1.99	0.2597	2.06	2.05	-0.004	2.06	-0.002

59	2.2	0.22	3.36	1.89	0.2435	2.15	2.26	0.018	2.06	-0.018
67	2.12	0.11	3.56	2.07	0.2785	1.86	1.95	0.025	1.99	0.032
69	2.14	0.07	3.58	2.09	0.2742	1.86	1.86	0.002	2.00	0.033
68	2.17	0.07	3.5	1.99	0.2655	1.94	1.91	-0.008	2.03	0.019
18	2.25	0.16	3.54	2.05	0.2557	1.96	2.09	0.025	2.08	0.023
66	2.25	0.06	3.61	2.09	0.2679	1.86	1.86	0.001	2.08	0.045
65	2.17	0.08	3.67	2.2	0.2625	1.84	1.84	0.000	2.00	0.035
6	2.25	0.1	3.68	2.22	0.2355	1.95	1.88	-0.015	2.05	0.018
60	2.19	0.12	3.55	1.94	0.2531	1.96	2.09	0.025	2.07	0.021
8	2.24	0.1	3.69	2.17	0.2403	1.92	1.92	0.001	2.06	0.025
26	2.27	0.03	3.89	2.37	0.2369	1.79	1.71	-0.020	2.06	0.048
14	2.18	0.06	3.74	2.08	0.2536	1.83	1.93	0.022	2.05	0.044
11	2.23	0.04	3.92	2.35	0.2297	1.80	1.76	-0.009	2.04	0.043
28	2.3	0.03	3.95	2.39	0.2165	1.84	1.72	-0.025	2.08	0.039
70	2.29	0.11	3.88	2.23	0.2021	1.98	1.99	0.002	2.11	0.018
53	2.38	0.46	3.64	1.99	0.131	3.04	2.92	-0.007	2.20	-0.070
29	2.28	0.06	4.03	2.4	0.2213	1.75	1.82	0.014	2.08	0.053
61	2.41	0.37	3.76	2.11	0.143	2.68	2.67	-0.001	2.20	-0.044
22	2.28	0.04	4.03	2.4	0.2072	1.81	1.77	-0.009	2.08	0.039
30	2.31	0.09	4.08	2.54	0.1887	1.88	1.81	-0.011	2.08	0.026
54	2.35	0.51	3.69	2.01	0.1146	3.31	3.03	-0.013	2.18	-0.083
52	2.4	0.44	3.71	1.97	0.1278	3.00	2.94	-0.004	2.23	-0.061
9	2.38	0.28	3.82	2.07	0.1589	2.40	2.51	0.009	2.20	-0.021
27	2.34	0.06	4.15	2.51	0.1903	1.80	1.79	-0.001	2.11	0.040
62	2.48	0.44	3.84	2.15	0.1089	3.21	2.88	-0.016	2.25	-0.063
80	2.28	0	4.34	2.7	0.2235	1.48	1.61	0.030	2.06	0.091
23	2.34	0.05	4.18	2.5	0.188	1.79	1.79	0.000	2.12	0.042
64	2.37	0.27	3.98	2.19	0.1434	2.40	2.46	0.004	2.19	-0.019
75	2.38	0.18	4.07	2.37	0.1442	2.27	2.15	-0.012	2.16	-0.011
57	2.35	0.27	3.99	2.13	0.15	2.31	2.51	0.016	2.19	-0.011
58	2.35	0.27	4	2.16	0.1454	2.35	2.49	0.010	2.18	-0.015
63	2.47	0.41	3.97	2.19	0.0937	3.40	2.83	-0.024	2.25	-0.064
5	2.32	0	4.46	2.85	0.1973	1.48	1.56	0.019	2.07	0.080
51	2.38	0.16	4.19	2.42	0.1696	1.89	2.11	0.024	2.16	0.029
17	2.36	0.06	4.3	2.57	0.1807	1.71	1.81	0.016	2.13	0.050
50	2.38	0.11	4.22	2.43	0.1735	1.84	2.00	0.020	2.17	0.036
21	2.35	0.06	4.32	2.62	0.1761	1.72	1.79	0.011	2.12	0.046
43	2.49	0.37	4.08	2.34	0.1118	2.76	2.66	-0.005	2.24	-0.034
16	2.41	0.27	4.06	2.24	0.1258	2.54	2.46	-0.005	2.21	-0.025
2	2.31	0	4.42	2.72	0.1989	1.51	1.63	0.024	2.08	0.078

49	2.38	0.1	4.24	2.51	0.156	1.95	1.93	-0.003	2.15	0.020
48	2.42	0.14	4.32	2.55	0.1632	1.81	2.03	0.026	2.18	0.038
45	2.55	0.35	4.17	2.43	0.0927	3.03	2.61	-0.020	2.27	-0.041
20	2.47	0.14	4.23	2.41	0.1309	2.23	2.10	-0.011	2.23	0.000
44	2.53	0.4	4.24	2.52	0.0885	3.01	2.69	-0.014	2.25	-0.040
71	2.47	0.21	4.25	2.48	0.1089	2.53	2.23	-0.020	2.22	-0.021
42	2.56	0.4	4.24	2.49	0.0719	3.61	2.73	-0.029	2.27	-0.055
1	2.33	0	4.66	2.91	0.1821	1.35	1.60	0.047	2.09	0.089
47	2.41	0.13	4.47	2.64	0.1402	1.81	2.01	0.019	2.17	0.031
41	2.55	0.38	4.37	2.62	0.0834	2.90	2.64	-0.010	2.26	-0.031
46	2.57	0.45	4.32	2.57	0.0677	3.59	2.84	-0.022	2.27	-0.051
79	2.35	0	4.69	2.96	0.1769	1.34	1.58	0.045	2.09	0.087
12	2.38	0.03	4.6	2.81	0.1546	1.55	1.71	0.022	2.13	0.057
73	2.47	0.23	4.42	2.61	0.1021	2.37	2.26	-0.007	2.21	-0.010
72	2.39	0.06	4.61	2.73	0.1508	1.56	1.83	0.033	2.16	0.056
31	2.51	0.13	4.62	2.8	0.0835	2.36	1.98	-0.022	2.22	-0.007
10	2.45	0.06	4.73	3	0.1056	1.78	1.72	-0.005	2.16	0.024
24	2.57	0.08	4.69	2.94	0.0912	2.07	1.80	-0.019	2.25	0.009
74	2.64	0.24	4.6	2.77	0.0586	3.25	2.29	-0.030	2.31	-0.029
33	2.55	0.12	4.78	3.23	0.069	2.33	1.79	-0.029	2.19	-0.006
13	2.47	0.05	4.73	2.89	0.1056	1.78	1.76	-0.002	2.19	0.025
4	2.39	0	4.91	3.1	0.1539	1.19	1.58	0.059	2.12	0.089
40	2.61	0.15	4.69	2.73	0.0612	2.86	2.11	-0.028	2.31	-0.018
25	2.57	0.08	4.88	3.05	0.092	1.68	1.81	0.009	2.25	0.029
32	2.57	0.13	4.77	2.8	0.0612	2.61	2.04	-0.023	2.28	-0.011
34	2.54	0.13	4.79	2.67	0.0624	2.52	2.12	-0.015	2.29	-0.008
19	2.5	0.06	4.94	3.12	0.0569	2.25	1.73	-0.023	2.19	-0.002
35	2.56	0.12	5	3.13	0.0313	3.36	1.90	-0.032	2.24	-0.020
38	2.54	0.18	5.13	3.13	0.039	2.21	2.10	-0.003	2.24	0.000
39	2.62	0.15	5.11	3.1	0.0225	3.75	2.04	-0.023	2.30	-0.017
78	2.49	0	5.34	3.51	0.0973	0.76	1.52	0.067	2.17	0.078
15	2.53	0.07	5.2	3.17	0.0412	1.81	1.82	0.000	2.23	0.009
36	2.61	0.15	5.23	3.26	0.0264	2.40	2.00	-0.007	2.27	-0.002
37	2.57	0.07	5.23	3.09	0.0312	2.12	1.87	-0.005	2.28	0.002
76	2.52	0	5.42	3.55	0.0746	0.66	1.53	0.057	2.19	0.063
3	2.53	0	5.52	3.6	0.0636	0.42	1.53	0.060	2.20	0.062

Col. 8-11 has shown remarkable agreement with the ω_p values for zero difference in porosity but the option (a) of defining $\omega_p = (V_p/V_s) + \rho_b * C_{frac}$ is undoubtedly the better. Option (b) may also take variations such as –

$$\omega_p = \frac{(\rho_b + V_p/V_s)}{2}$$

and the suitable algorithm for a specific location or formation be fixed through appropriate core studies.

It becomes therefore apparent that in combination with the density log and a clay indicator such as GR, P-wave velocity can be used for deriving precise porosity values. Modified RHG equation as above thus provide for a single, continuous and smooth behavior in the wide range of porosity and clay content and allows tailoring to any particular environment.

IV. Application for S-Wave Velocity

Modified RHG equation can be applied to the S-wave velocity also by choosing Δt_{ma} appropriately for the shear wave and retaining the p-wave travel time of the fluid. ω values tend to be show significant scatter in the case of shaly sands. Modified RHG algorithm thus presents ample scope for developing ω_p and ω_s as parameters capable of offering shaly sand characterization. Table-2 presents relevant data for few samples:

Sample No	ρ	Clay Fraction	V_p	V_s	Porosity	ω_s for 0 Diff: in ϕ
1	2	3	4	5	6	7
56	2.12	0.12	3.17	1.77	0.2945	2.58
55	2.09	0.11	3.2	1.75	0.2993	2.57
7	2.24	0.16	3.36	1.99	0.2597	2.58
59	2.2	0.22	3.36	1.89	0.2435	2.80
67	2.12	0.11	3.56	2.07	0.2785	2.40
69	2.14	0.07	3.58	2.09	0.2742	2.41
68	2.17	0.07	3.5	1.99	0.2655	2.55
18	2.25	0.16	3.54	2.05	0.2557	2.55
66	2.25	0.06	3.61	2.09	0.2679	2.44
65	2.17	0.08	3.67	2.2	0.2625	2.36
6	2.25	0.1	3.68	2.22	0.2355	2.51
60	2.19	0.12	3.55	1.94	0.2531	2.68
8	2.24	0.1	3.69	2.17	0.2403	2.53
26	2.27	0.03	3.89	2.37	0.2369	2.34
14	2.18	0.06	3.74	2.08	0.2536	2.54
11	2.23	0.04	3.92	2.35	0.2297	2.41
28	2.3	0.03	3.95	2.39	0.2165	2.46
70	2.29	0.11	3.88	2.23	0.2021	2.77

When the clay content is less, S-wave velocity could be used to have porosity with $\omega_s \approx 2.5$ and the physical expression for the same remains to be explored.

V. Conclusions

A universal algorithm that relates P and S-wave travel times to porosity has been suggested for petrophysical applications. Working of the algorithm in the few samples examined suggests that the model may be customized easily to shaly sands and other hydrocarbon bearing formations. Presented algorithm provide for a single, continuous and smooth behavior in the wide range of porosity and clay content and allows tailoring to any particular environment. Method offers single parameter characterization of the velocity-porosity transform while other empirical models demand more number of curve fitting parameters.

In combination with the density log and a clay indicator such as GR, P-wave velocity can be used for deriving precise porosity values.

VI. References

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