Chapter 2

AVO THEORY

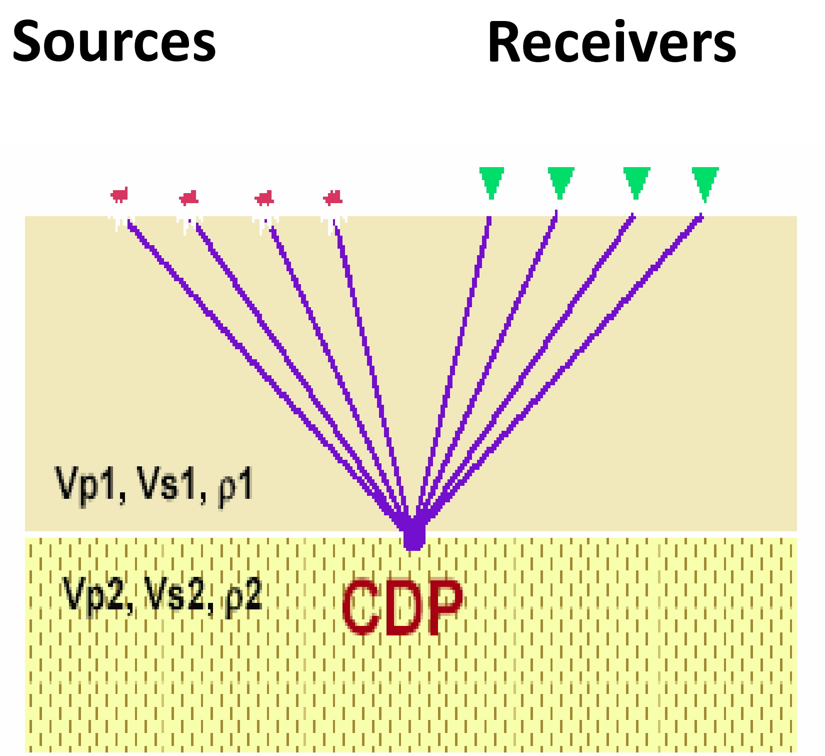
AVO in Seismic Data

- AVO analysis is an extension of seismic amplitude analysis to include pre-stack data
- Amplitude analysis became possible in 1970’s when seismic data was processed without AGC correction
    - Gas identified as high amplitude ‘bright spots’
    - Coal, tuning, basalt, carbonates, overpressure etc. all give brightspots too so AVO needed to differentiate
- Ostrander (1984) first to promote the use of AVO

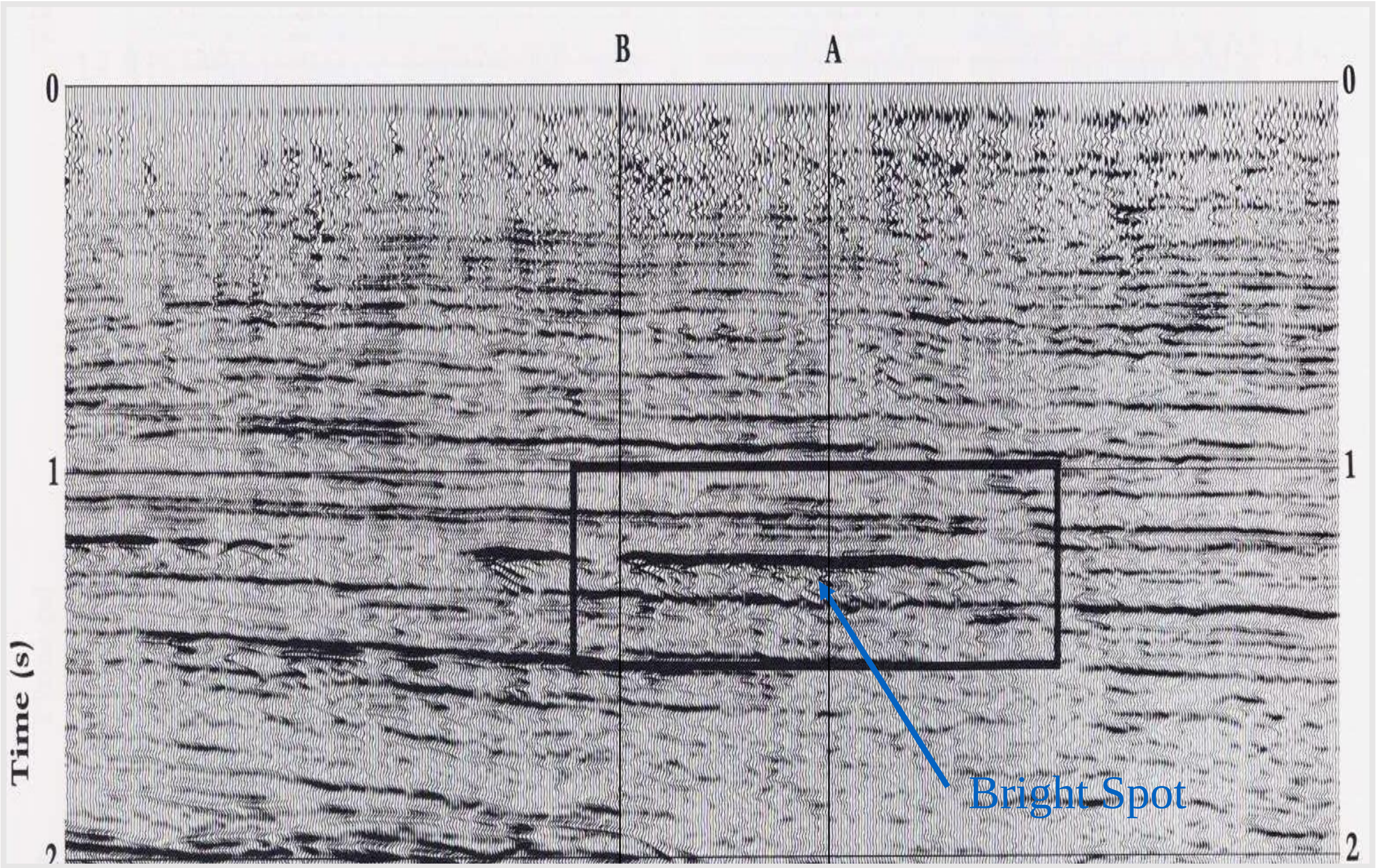
2D & 3D Amplitude versus Offset (AVO)

CONCEPT

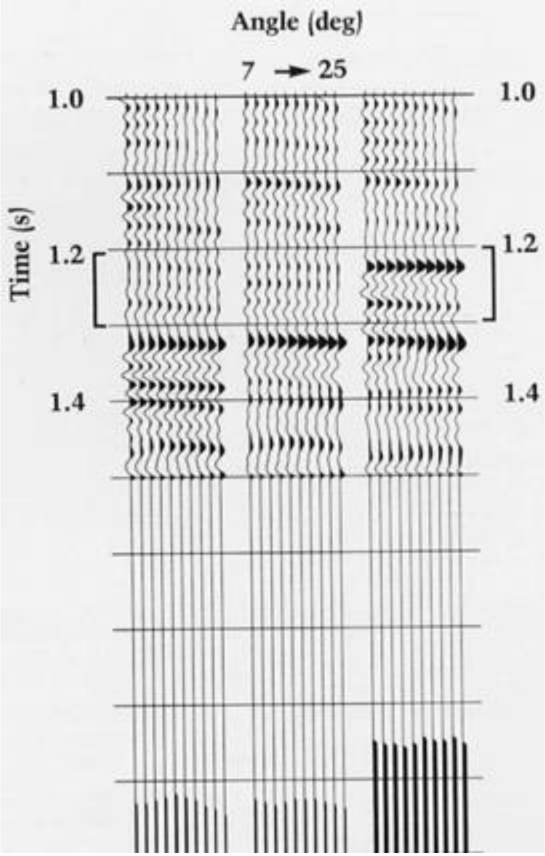
- AVO analyzes changes in amplitude of seismic waves that are dependent on source-receiver distance
- AVO deals with pre-stack seismic attributes for hydrocarbon discrimination

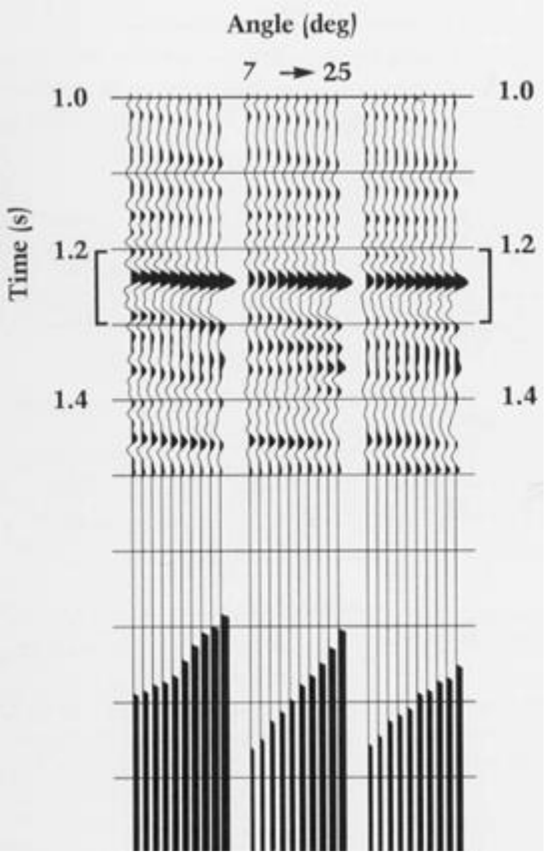


An Example: Stacked Section



AVO Example

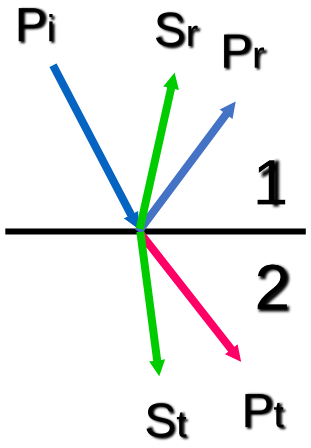




CMP gathers from positions B & A respectively. Both the height & thickness of the bar graphs at the bottom are proportional to the relative RMS amplitude within the window at 1.2 to 1.3s. At location B, the increase in amplitude with angle within the bright spot is consistent with the known presence of gas at this level in the survey area. At location A, amplitude variations are small.

Elastic Waves at a Plane Interface

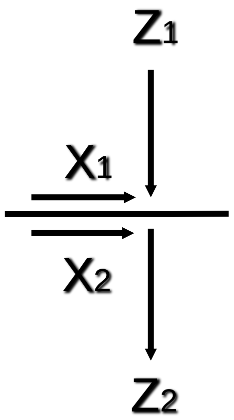
Ray Paths



Snell's Law
Obeyed

Continuous Properties

Displacements

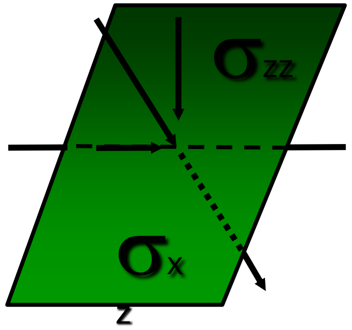


$X_1 = X_2$

$Y_1 = Y_2$

$Z_1 = Z_2$

Vertical Stresses



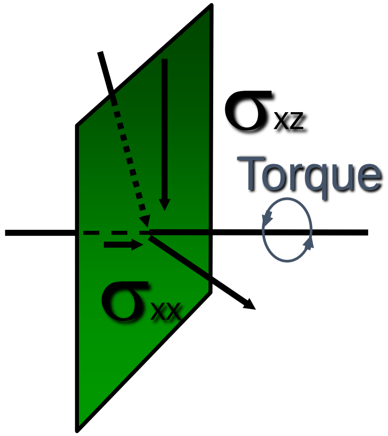
$\sigma_{xz1} = \sigma_{xz2}$

$\sigma_{yz1} = \sigma_{yz2}$

$\sigma_{zz1} = \sigma_{zz2}$

Discontinuous Properties

Horizontal Stresses

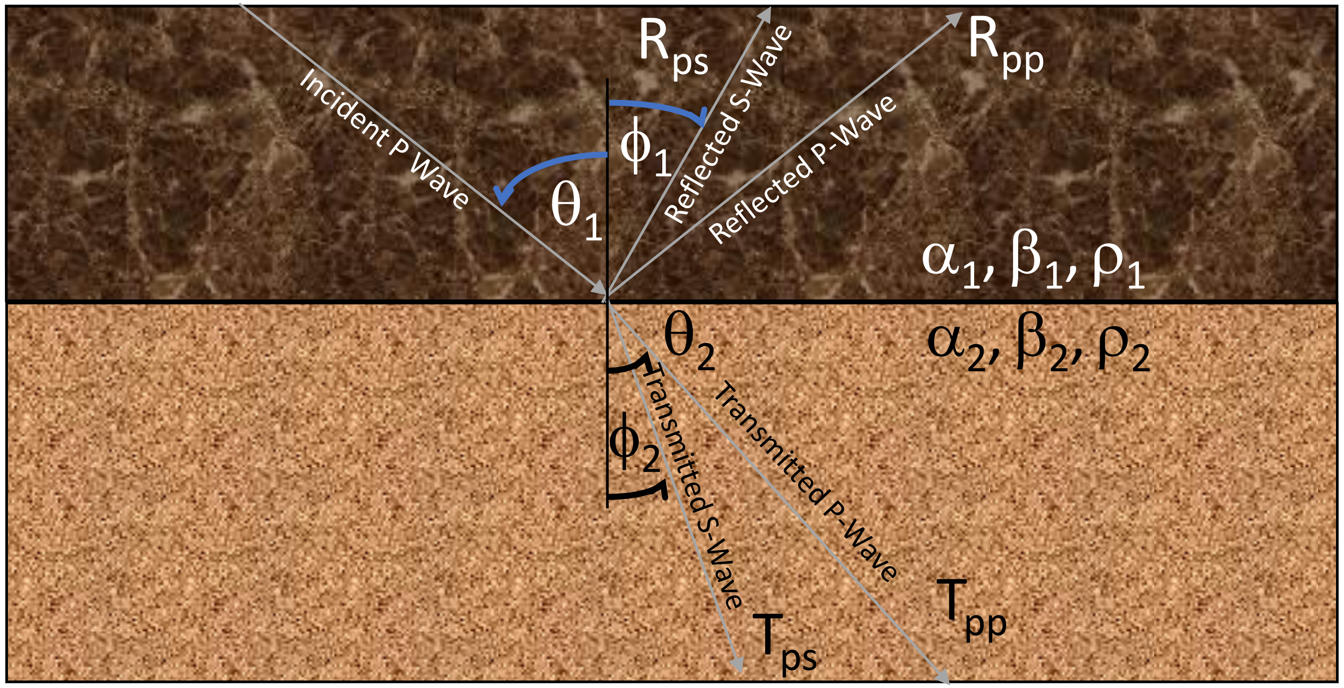


$\sigma_{xx1} \neq \sigma_{xx2}$

$\sigma_{yy1} \neq \sigma_{yy2}$

$\sigma_{xy1} \neq \sigma_{xy2}$

Snell's Law



Ray geometry determined by Snell's Law, derived from Fermat's principle that the ray-paths define the minimum travel time.

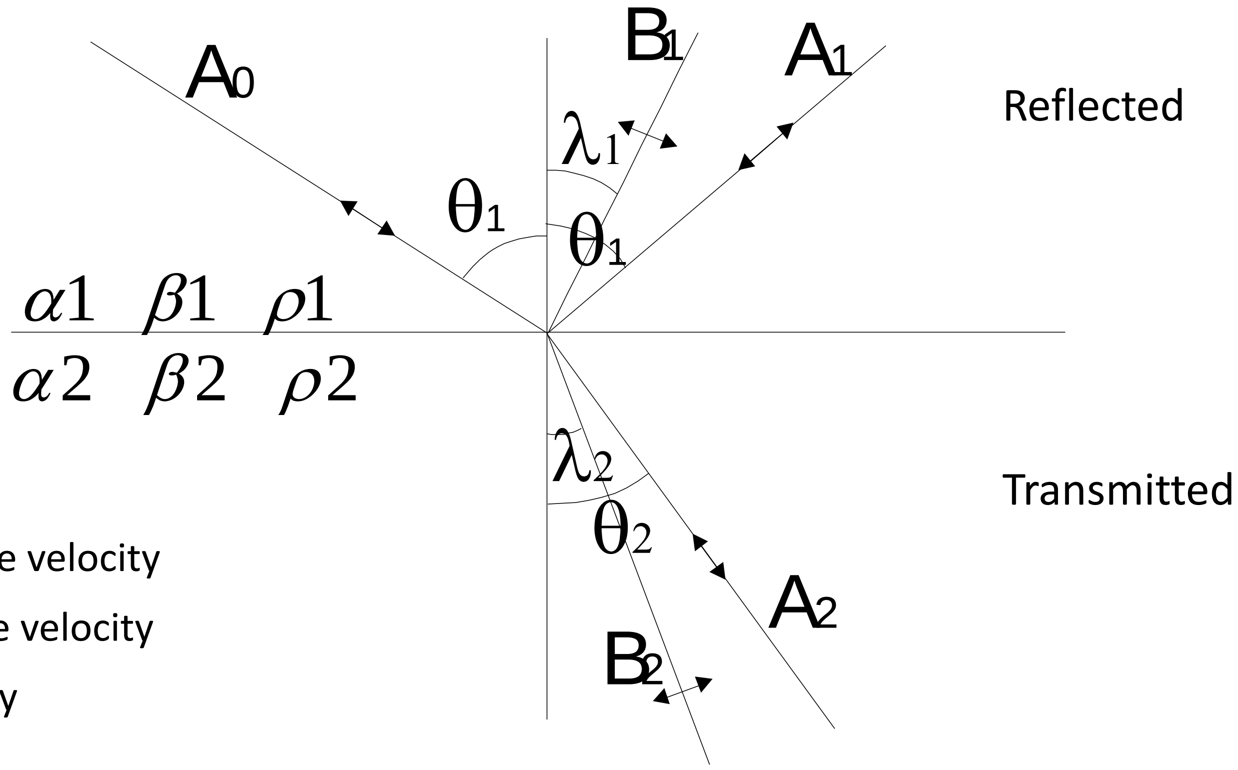
$$\frac{\sin(\theta_1)}{\alpha_1} = \frac{\sin(\theta_2)}{\alpha_2} = \frac{\sin(\phi_1)}{\beta_1} = \frac{\sin(\phi_2)}{\beta_2}$$

Mathematical Foundation
Zoeppritz Equations

One of the basic assumptions about seismic data is that the seismic wave strikes the rock layer at vertical incidence.
In this case, the reflection coefficient is given as the following equation:

$$R_i = \frac{AI_{i+1} - AI_i}{AI_{i+1} + AI_i} \qquad (1)$$

Mathematical Foundation
Zoeppritz Equations



α = P-wave velocity

β = S-wave velocity

ρ = Density

Illustration of how the P-wave strike the boundary and split into 4 waves when it strikes at non zero incidence angle

Mathematical Foundation
Zoeppritz Equations (1919)

$$\begin{bmatrix} \sin\lambda_r & \cos\phi_r & \sin\lambda_r & \cos\phi_t \\ -\cos\lambda_r & \sin\phi_r & \cos\lambda_t & -\sin\phi_t \\ \sin 2\lambda_r & \dfrac{\alpha_1}{B_1}\cos 2\phi & \dfrac{\rho_2 B_2{}^2\,\alpha_1}{\rho_1 B_1{}^2\,\alpha_2}\sin 2\lambda_t & \dfrac{-\rho_2 B_2 \alpha_1}{\rho_1 B_1}\cos 2\phi_t \\ \cos 2\lambda_r & \dfrac{\beta_1}{\alpha_1}\sin 2\lambda_r & \dfrac{-\rho_2 \lambda_2}{\rho_1 B_1{}^2\,\alpha_2}\cos 2\phi & \dfrac{-\rho_2 B_2}{\rho_1\,\alpha_1}\sin 2\phi \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} -\sin\lambda_r \\ -\cos\lambda_r \\ \sin 2\lambda_r \\ -\cos 2\phi_t \end{bmatrix}$$

(3)

This equation (3) gives the final form of the Zoeppritz equation, and relates to the rays shown in previous figure.

P-P Reflection Coefficient Formula

Exact Formula (Aki & Richards, 1980; Mallick, 1993):

$$R_{PP} = \frac{A + Bp^2 + Cp^4 - Dp^6}{E + Fp^2 + Gp^4 + Dp^6},$$

$$p = \frac{\sin\theta_1}{\alpha_1} = \frac{\sin\phi_1}{\beta_1} = \frac{\sin\theta_2}{\alpha_2} = \frac{\sin\phi_2}{\beta_2}.$$

$$A = \left(\rho_2 \frac{\cos\theta_1}{\alpha_1} - \rho_1 \frac{\cos\theta_2}{\alpha_2}\right)\left(\rho_2 \frac{\cos\phi_1}{\beta_1} + \rho_1 \frac{\cos\phi_2}{\beta_2}\right),$$

$$B = -4\Delta\mu\left(\rho_2 \frac{\cos\theta_1}{\alpha_1}\frac{\cos\phi_1}{\beta_1} + \rho_1 \frac{\cos\theta_2}{\alpha_2}\frac{\cos\phi_2}{\beta_2}\right) - (\Delta\rho)^2 + 4(\Delta\mu)^2 \frac{\cos\theta_1}{\alpha_1}\frac{\cos\theta_2}{\alpha_2}\frac{\cos\phi_1}{\beta_1}\frac{\cos\phi_2}{\beta_2},$$

$$C = 4(\Delta\mu)^2\left(\frac{\cos\theta_1}{\alpha_1}\frac{\cos\phi_1}{\beta_1} - \frac{\cos\theta_2}{\alpha_2}\frac{\cos\phi_2}{\beta_2}\right) + 4\Delta\mu\Delta\rho,$$

$$D = 4(\Delta\mu)^2,$$

$$E = \left(\rho_2 \frac{\cos\theta_1}{\alpha_1} + \rho_1 \frac{\cos\theta_2}{\alpha_2}\right)\left(\rho_2 \frac{\cos\phi_1}{\beta_1} + \rho_1 \frac{\cos\phi_2}{\beta_2}\right),$$

$$F = -4\Delta\mu\left(\rho_2 \frac{\cos\theta_1}{\alpha_1}\frac{\cos\phi_1}{\beta_1} - \rho_1 \frac{\cos\theta_2}{\alpha_2}\frac{\cos\phi_2}{\beta_2}\right) + (\Delta\rho)^2 + 4(\Delta\mu)^2 \frac{\cos\theta_1}{\alpha_1}\frac{\cos\theta_2}{\alpha_2}\frac{\cos\phi_1}{\beta_1}\frac{\cos\phi_2}{\beta_2},$$

$$G = 4(\Delta\mu)^2\left(\frac{\cos\theta_1}{\alpha_1}\frac{\cos\phi_1}{\beta_1} + \frac{\cos\theta_2}{\alpha_2}\frac{\cos\phi_2}{\beta_2}\right) - 4\Delta\mu\Delta\rho,$$

$$\Delta\mu = \mu_2 - \mu_1 = \rho_2\beta_2^2 - \rho_1\beta_1^2;\ \Delta\rho = \rho_2 - \rho_1.$$

How do we simplify?

- Use average properties and changes in properties
- Assume changes in properties are small compared to average properties.

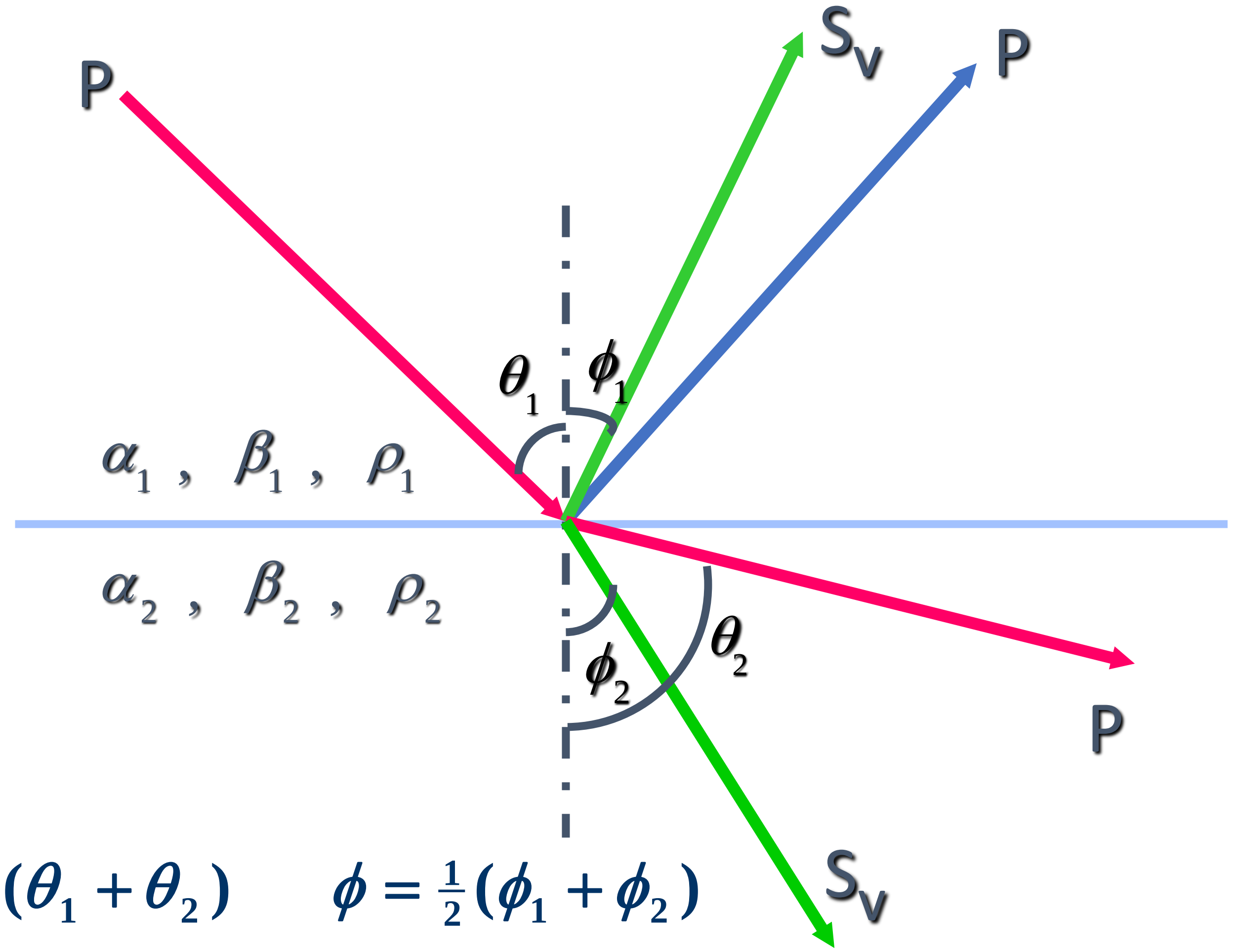
$$\alpha = \tfrac{1}{2}(\alpha_1 + \alpha_2) \qquad \beta = \tfrac{1}{2}(\beta_1 + \beta_2) \qquad \rho = \tfrac{1}{2}(\rho_1 + \rho_2)$$

$$\Delta\alpha = \alpha_2 - \alpha_1 \qquad \Delta\beta = \beta_2 - \beta_1 \qquad \Delta\rho = \rho_2 - \rho_1$$

- The average properties define a **background medium**

Approximation to the Zoeppritz Equations:

Use average angles



$$\theta = \tfrac{1}{2}(\theta_1 + \theta_2) \qquad \phi = \tfrac{1}{2}(\phi_1 + \phi_2)$$

Approximation to the Zoeppritz Equations:

The Aki, Richard and Frasier Approximation

$$R(\theta) = a\frac{\Delta\alpha}{\alpha} + b\frac{\Delta\rho}{\rho} + c\frac{\Delta\beta}{\beta} \qquad (4)$$

Rearrangement with three terms: Vp, Vs and density

where :

$$a = 1/(2\cos^2\theta) = 1/2 + \tan^2\theta$$
$$b = 0.5 - [(2\beta^2/\alpha^2)\sin 2\theta]$$
$$c = -(4\beta^2/\alpha^2)\sin^2\theta$$
$$\alpha = (\alpha_1 + \alpha_2)/2$$
$$\beta = (\beta_1 + \beta_2)/2$$
$$\rho = (\rho_1 + \rho_2)/2$$
$$\Delta\alpha = \alpha_2 - \alpha_1$$
$$\Delta\beta = \beta_2 - \beta_1$$
$$\Delta\rho = \rho_2 - \rho_1$$
$$\theta = (\theta_i + \theta_t)/2$$
$$\theta_t = \arcsin[(\alpha 2/\alpha 1)\sin\theta_i]$$

Approximation to the Zoeppritz Equations:
The Smith/Gidlow Method

Smith and Gidlow rearranged equation (4) in the following way:

$$R(\theta) = \frac{1}{2}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right) - 2\frac{\beta^2}{\alpha^2}\left[2\frac{\Delta\beta}{\beta} + \frac{\Delta\rho}{\rho}\right]\sin^2\theta + \frac{1}{2}\frac{\Delta\alpha}{\alpha}\tan^2\theta \qquad (5)$$

They then chose to remove the dependency on density by using Gardner's equation :

$$\rho = c\,\alpha^{1/4} \qquad (6)$$

which can be differentiated to give :

$$\Delta\rho / \rho = \frac{1}{4}\frac{\Delta\alpha}{\alpha} \qquad (7)$$

Substituting equation (7) into (5), we can re-express Aki and Richard's equation as the following weighted sum of P- and S- wave velocity variations:

$$R(\theta) = a\frac{\Delta\alpha}{\alpha} + b\frac{\Delta\beta}{\beta} \qquad (8)$$

Rearrangement with two terms: Vp and Vs

$$\text{where} \quad a = \frac{5}{8} - \frac{\beta^2}{\alpha^2}\sin^2\theta + \frac{1}{2}\tan^2\theta$$

$$\text{and} \quad b = -4\frac{\beta^2}{\alpha^2}\sin^2\theta$$

Approximation to the Zoeppritz Equations:
The Smith/Gidlow Method

Once the P and S velocities have been extracted, they can be combined in various ways. The first is termed ‘ pseudo Poisson’s ratio’, and can be written :

$$\frac{\Delta\sigma}{\sigma} = \frac{\Delta\alpha}{\alpha} - \frac{\Delta\beta}{\beta} \qquad (9)$$

The second, termed ‘fluid factor’ is based on the ‘mudrock equation by Castagna :

(10)

Approximation to the Zoeppritz Equations:
The Smith/Gidlow Method

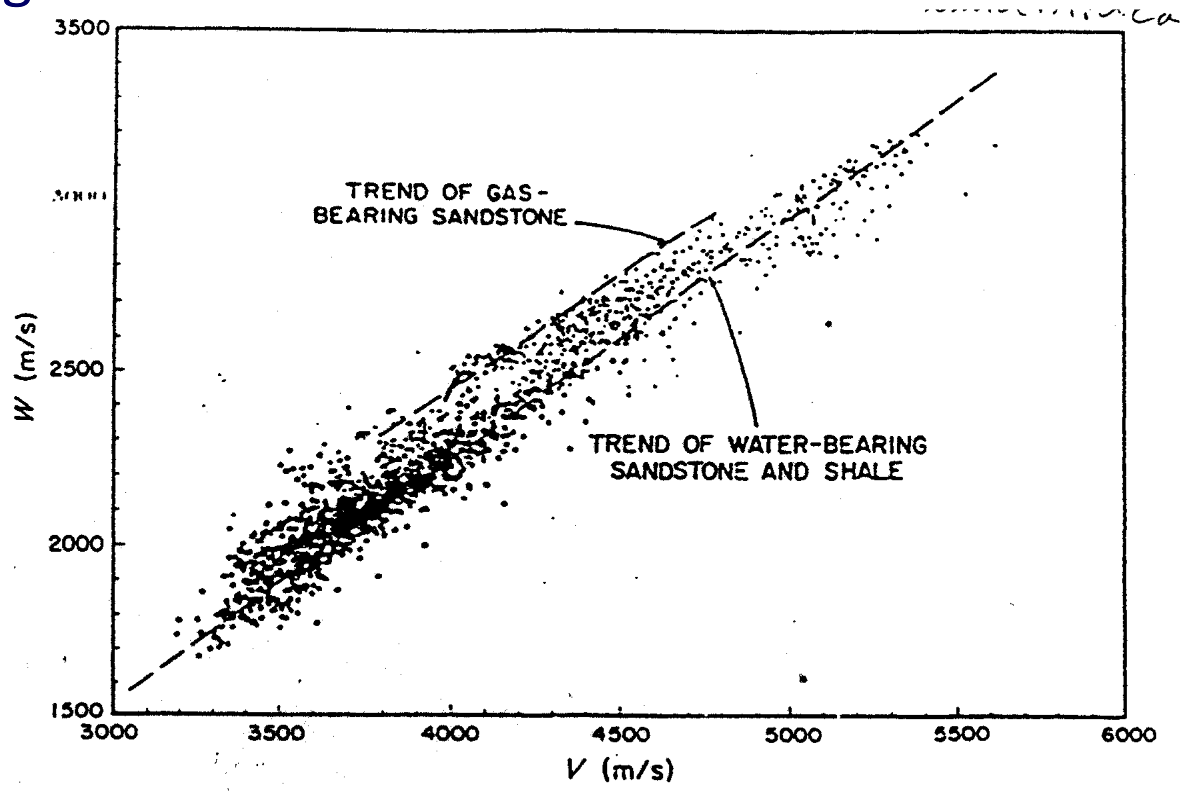
For a gas reservoir, we can define the “fluid factor” error from the following equation:

$$\Delta F = \frac{\Delta\alpha}{\alpha} - 1.16\frac{\beta}{\alpha}\frac{\Delta\beta}{\beta} \qquad (12)$$

In other words, if $\Delta F = 0$, the reservoir is non-prospective, but if $|\Delta F| \neq 0$, the reservoir is prospective.

Approximation to the Zoeppritz Equations:
The Smith/Gidlow Method

Real data example of the previous case for a line crossing an existing gas well :



The cross plot between P-velocity against S-velocity (Russel, 1998)

Approximations to the Zoeppritz equations:
Fatti et al. (1994)

- Starting point is same version of Aki & Richards (1980) that Smith & Gidlow (1987) use:
- However, unlike Smith & Gidlow assumption is that Gardner's relationship cannot be applied
- Instead they use P and S-wave acoustic impedances, Ip and Is

Approximation to the Zoeppritz Equations:
Shuey's Approximation

Shuey (1985) gave a closed form approximation of the Zoeppritz's equations, as follows :

$$R(\theta) = R_p + (R_p A_0 + \frac{\Delta\sigma}{(1-\sigma)^2})\sin^2\theta + 1/2\frac{\Delta\alpha}{\alpha}(\tan^2\theta - \sin^2\theta) \qquad (17)$$

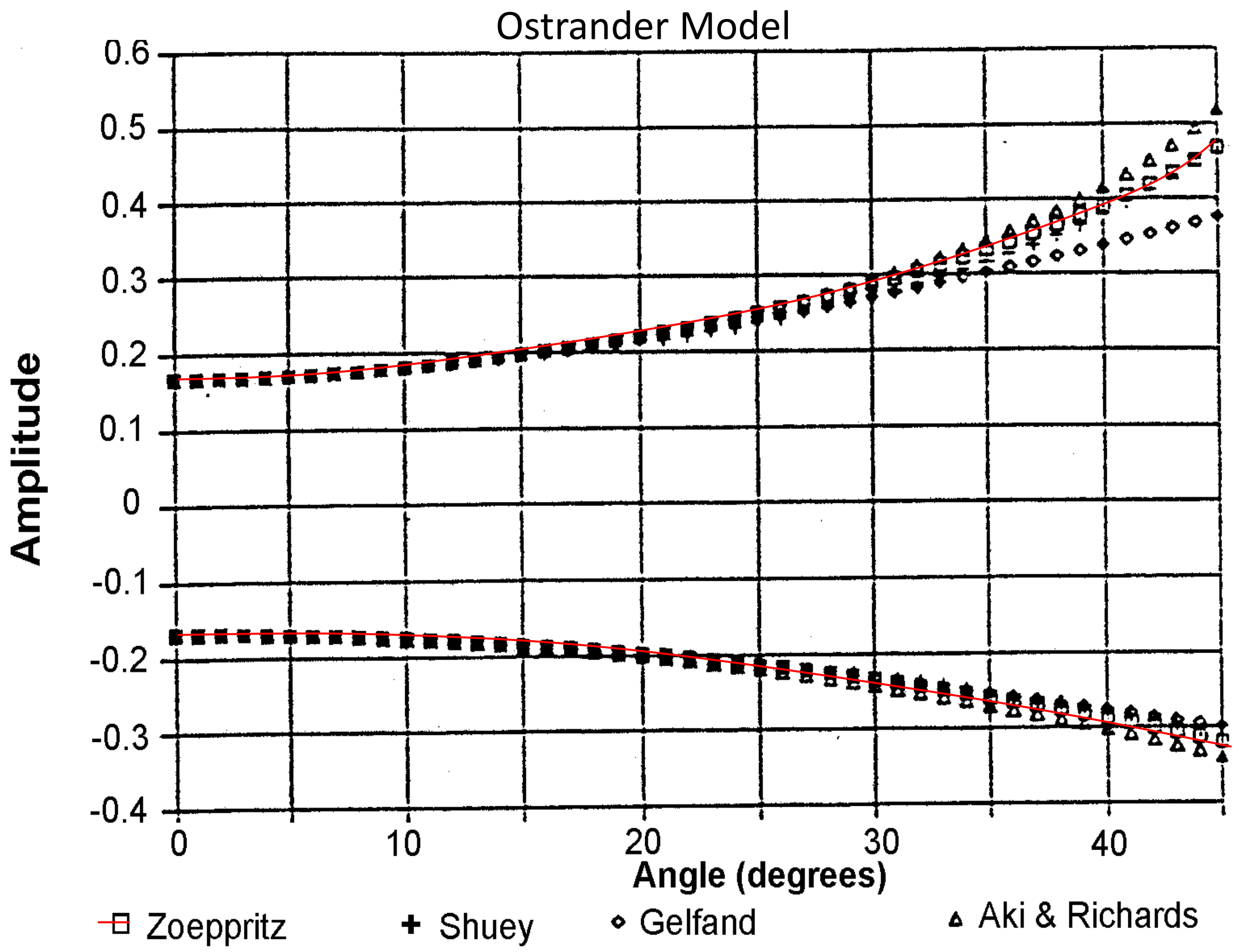
where $\sigma = (\sigma_1+\sigma_2)/2$ and $\Delta\sigma = \sigma_2 - \sigma_1$

$$A_0 = B - 2(1+B)\frac{1-2\sigma}{1-\sigma}$$

$$B = \frac{\Delta\alpha/\alpha}{\Delta\alpha/\alpha + \Delta\rho/\rho}$$

Previous approximations based on α, β and ρ....
.....Shuey's based on α, β and σ (Poisson's ratio)

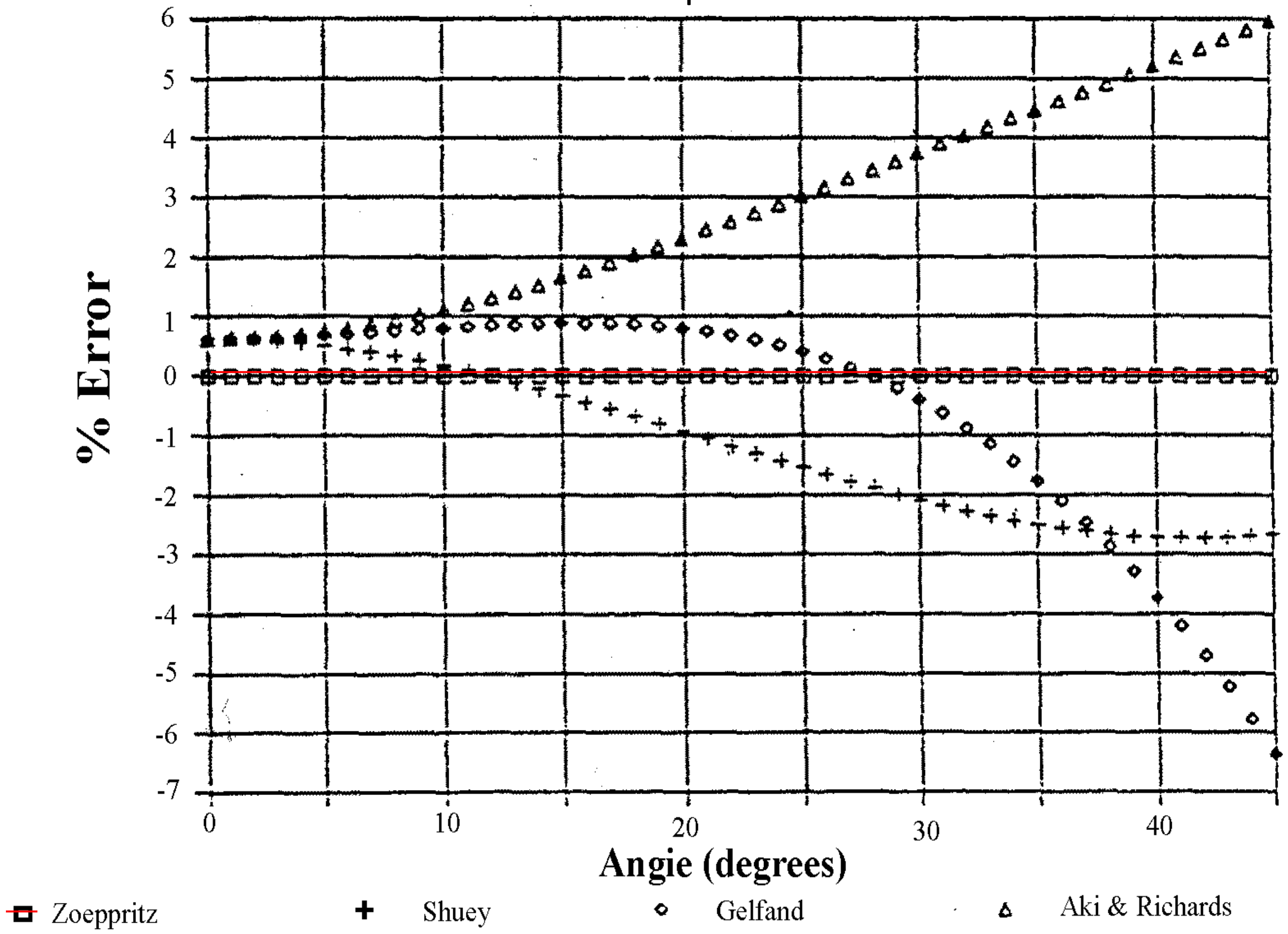
Approximation to the Zoeppritz's Equations



A comparison of Zoeppritz's equations and its approximations for a simple gas sand model (Russel, 1998)

Approximation to the Zoeppritz's Equations

Error at top interface



The error for the negative reflector shown in previous slide (Russel, 1998)

2-term AVO Analysis

- Shuey Equation (approximation of Aki & Richards AVO equation)

  $R(\theta) = P + G \sin^2(\theta)$

  $\quad = R_0 + (R_0 - 2R_s) \sin^2(\theta)$

- From AVO theory for angles < 30 degrees and $V_p = 2V_s$
  - Intercept, $P = R_0$
  - Gradient, $G = R_0 - 2R_s$
- AVO Attributes
  - Shear Reflectivity, $R_s = (P-G)/2$

Shuey 2-Term AVO attributes

- Intercept, P

$$R_0 = \frac{1}{2}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right) \qquad R_s = \frac{1}{2}\left(\frac{\Delta\beta}{\beta} + \frac{\Delta\rho}{\rho}\right)$$

- Poisson Ratio Contrast

$$\sigma = \frac{\alpha^2 - 2\beta^2}{2(\alpha^2 - \beta^2)}$$

$$\frac{\Delta\sigma}{\sigma} = \frac{4}{3}\left(\frac{\Delta\alpha}{\alpha} - \frac{\Delta\beta}{\beta}\right)$$

$$= \frac{8}{3}\left(R_0 - R_s\right)$$

$$= \frac{4}{3}\left(P + G\right)$$

Comparison of Linear (2-term) AVO Equations

- Assumptions
  - $\tan^2\theta = \sin^2\theta$ (OK for angles < 30 degrees)
  - $V_p = 2V_s$
- Shuey $R(\theta) = R_0 + (R_0 - 2R_s)\sin^2(\theta)$
- Smith $R(\theta) = R_0/\cos^2(\theta) + 2R_s\sin^2(\theta)$
- Hilterman $R(\theta) = R_0\cos^2(\theta) + 2(R_0 - R_s)\sin^2(\theta)$ (Hilterman, 2001)

$$R_0 = \frac{\rho_2\alpha_2 - \rho_1\alpha_1}{\rho_2\alpha_2 + \rho_1\alpha_1} \approx \frac{1}{2}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right)$$

$$R_s = \frac{\rho_2\beta_2 - \rho_1\beta_1}{\rho_2\beta_2 + \rho_1\beta_1} \approx \frac{1}{2}\left(\frac{\Delta\beta}{\beta} + \frac{\Delta\rho}{\rho}\right)$$

Approximations to the Zoeppritz equations

Pan & Gardner (1985)

- Starting from Shuey, Pan & Gardner introduce a P-Wave modulus M and a Shear wave modulus μ and rewrite in the form:

$$y = a + bx + cx^2$$

where

$$y = R\cos^2\theta$$
$$x = \sin^2\theta$$
$$a = \frac{1}{4}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta M}{M}\right)$$
$$b = -\frac{1}{2}\left(\frac{\Delta\rho}{\rho} + 4\frac{\Delta\mu}{M}\right)$$
$$c = 2\frac{\Delta\mu}{M}$$

AVO curve fit with parabola and solved for a,b and c.

Why 2-term AVO analysis?

- Robust on Noisy data
- Geometry limits useable angles to around 30 degrees
- Vp/Vs=2 assumption simplifies the interpretation

Limitations

- 3-term is more accurate but needs low noise and large angles!

Offset to Angle Transformation

Both Zoeppritz's and Shuey's equations are dependent upon the incidence angle of incidence at which the seismic ray strikes the horizon of interest.

Since the seismic data was recorded as a function of offset, the data must be transformed from the offset domain to the angle domain:

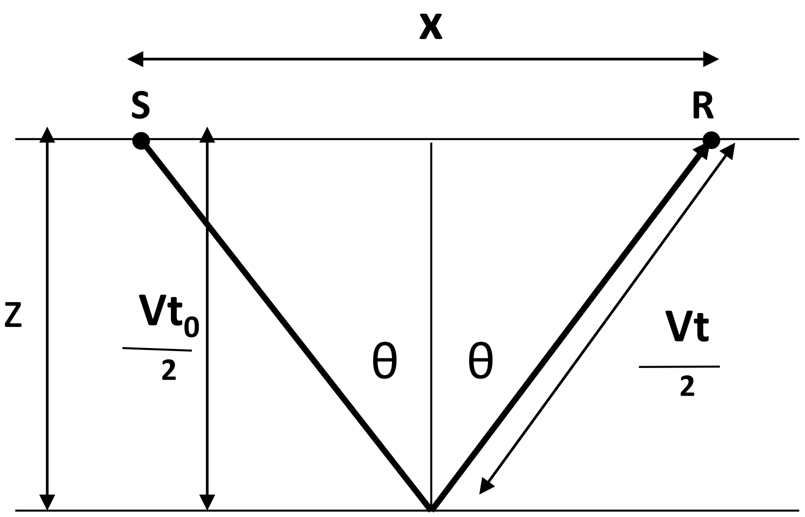
Offset to Angle Transformation

To transform from constant offset to constant angle, use the following equations :

$$\tan\theta = \frac{X}{2Z}$$

$$Z = \frac{Vt_0}{2}$$

where: θ = angle of incidence
X = offset
Z = depth
V = velocity (RMS or average)
t_0 = total zero offset travel time



Relationship between depth and offset

$$\tan\theta = \frac{X}{Vt_0}$$

$$\downarrow$$

$$X = Vt_0 \tan\theta$$

Effective angle of angle stack data

- Angle stacks can be inverted using the concept of the effective impedance, or **angle impedance**, at a constant angle of incidence.
- Using angle reflectivity and angle impedance logs computed for the effective angle of the angle stack makes the application of the convolution model valid for inversion of the angle stacks
- The effective angle of the angle stack is the single angle corresponding to the unweighted arithmetic mean of the reflection coefficient over the traces in the angle stack.
- As the reflection coefficient according to Shuey's approximation is approximately linear related to sin^2(q), the effective angle, qeff, is given by:

$$\sin^2(\theta_{eff}) = \frac{\sin^2(\theta_{\min}) + \sin^2(\theta_{\max}) + \sin(\theta_{\min})\sin(\theta_{\max})}{3}$$

- θ_{min} and θ_{max} are minimum and maximum angle of the angle stack
- The effective angle can be up to 16% larger than the arithmetic mean of the minimum and maximum angle of the angle stack.

Approximation of Zoeppritz

Shuey's equation

Once we have transformed from offset to angle, we can use the Shuey's approximation, which is written :

$$R(\theta) = Rp + G \sin^2\theta \qquad (32)$$

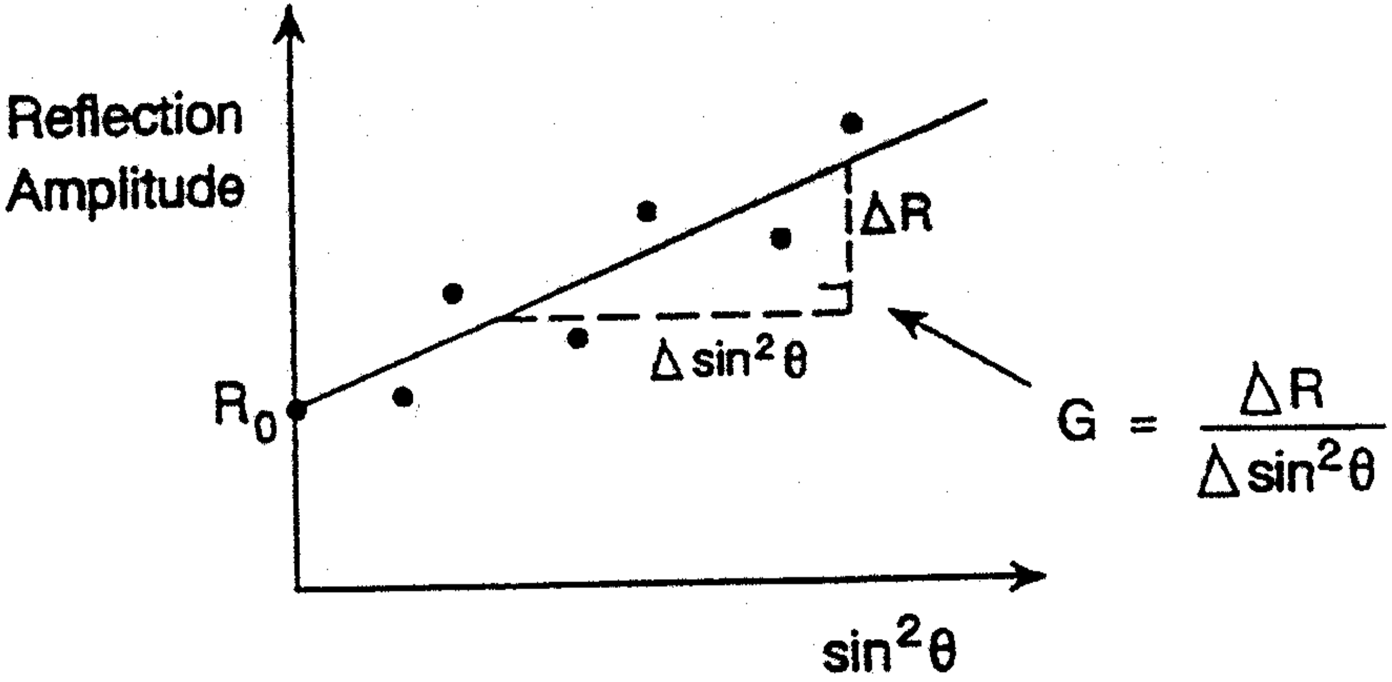
where $R(\theta)$ = change of reflection coefficient with angle θ

Rp = P-wave reflection coefficient at normal incidence

G = gradient term depending on change of Poisson's ratio

An example of this curve plot is shown on next figure

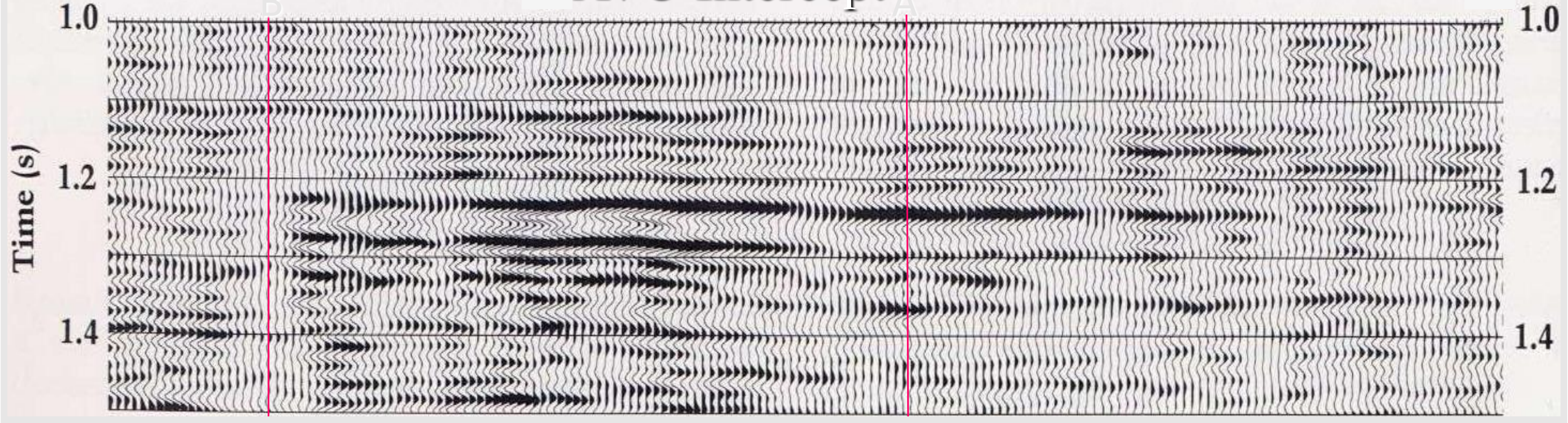
Approximation of Zoeppritz
Shuey's equation



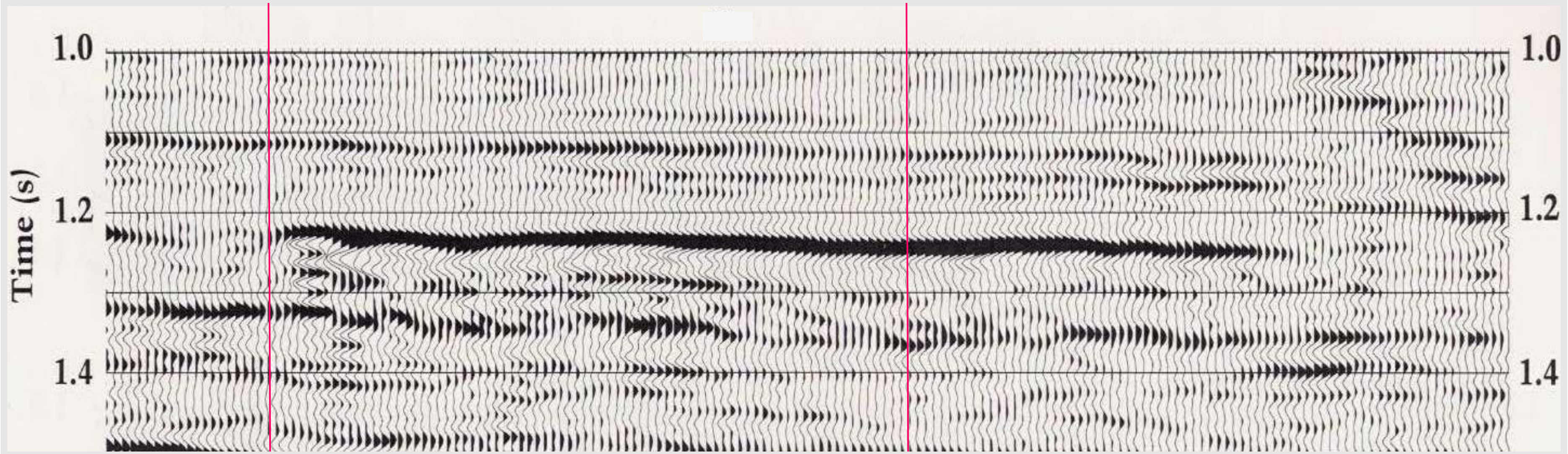
Example of the plot of amplitude versus $\sin^2\theta$

Standard AVO Products

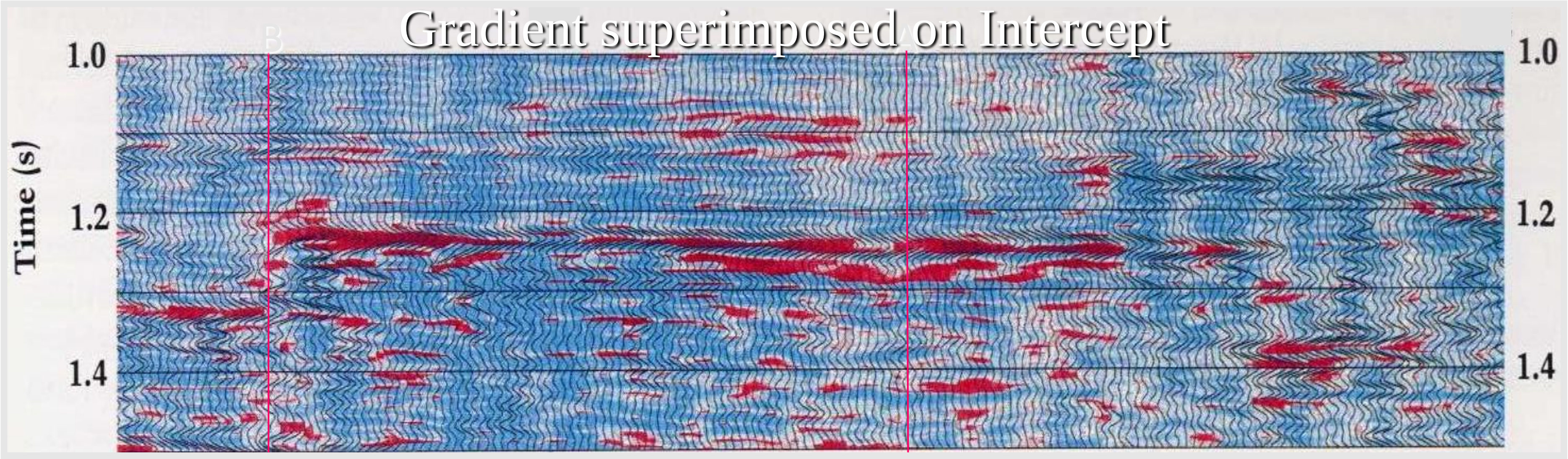
AVO Intercept



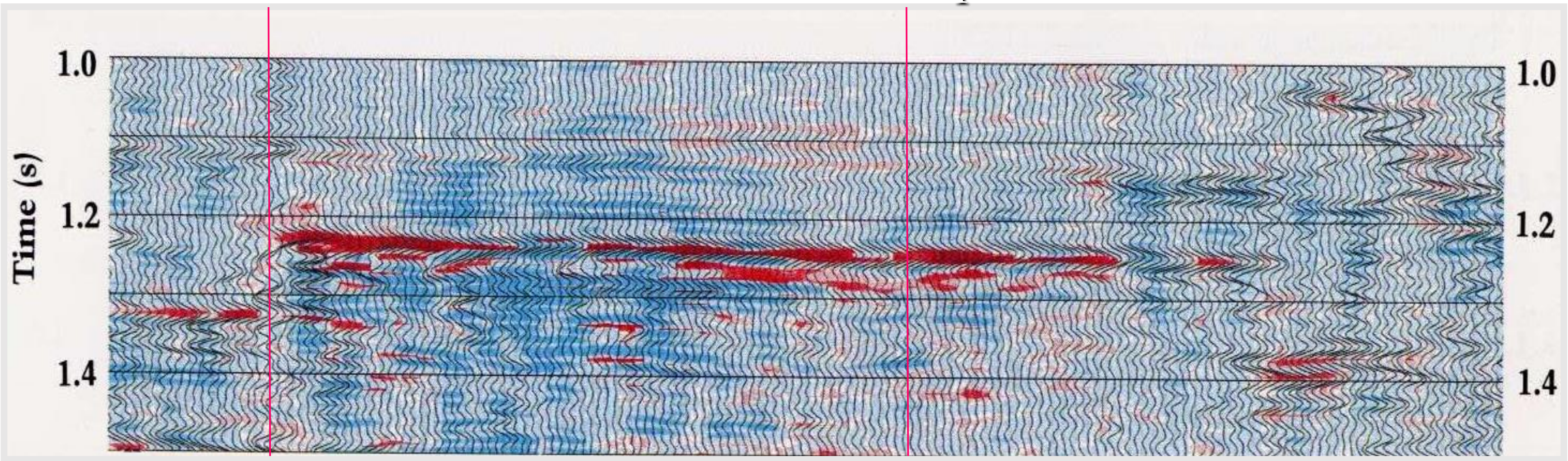
Conventional Stack



Gradient Displays



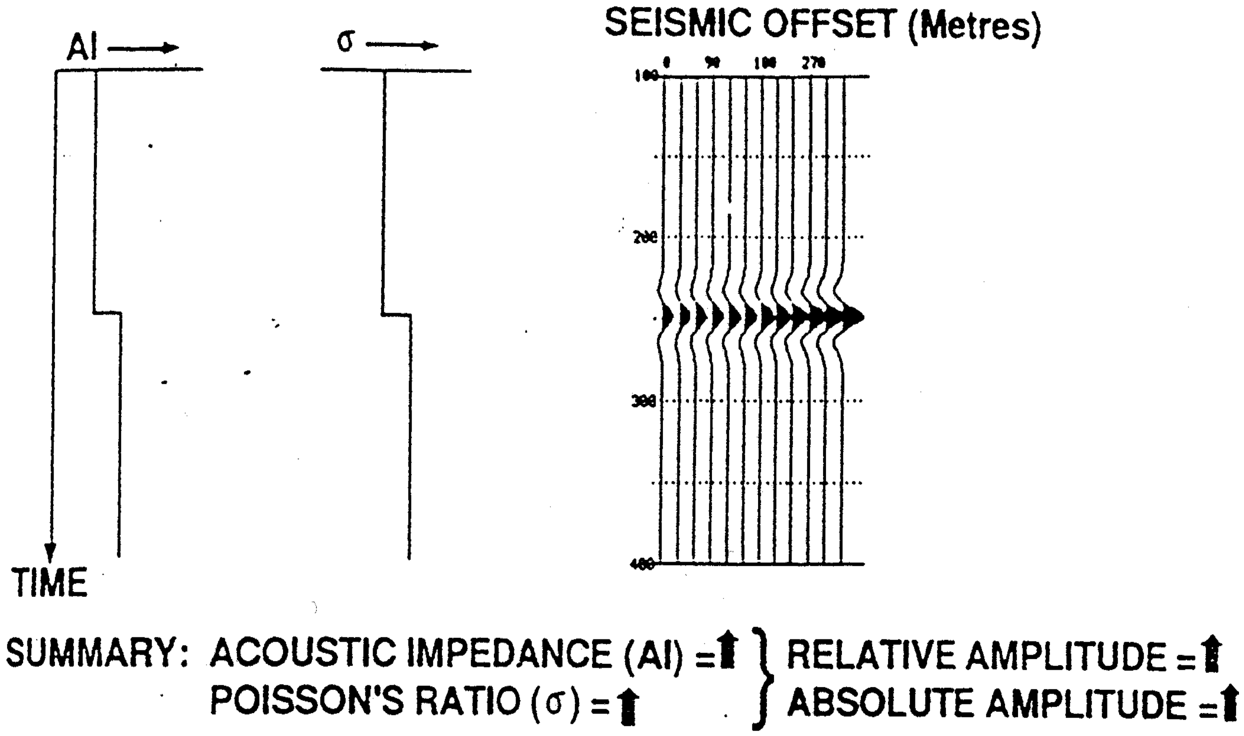
Gradient * Intercept



Limits of approximations

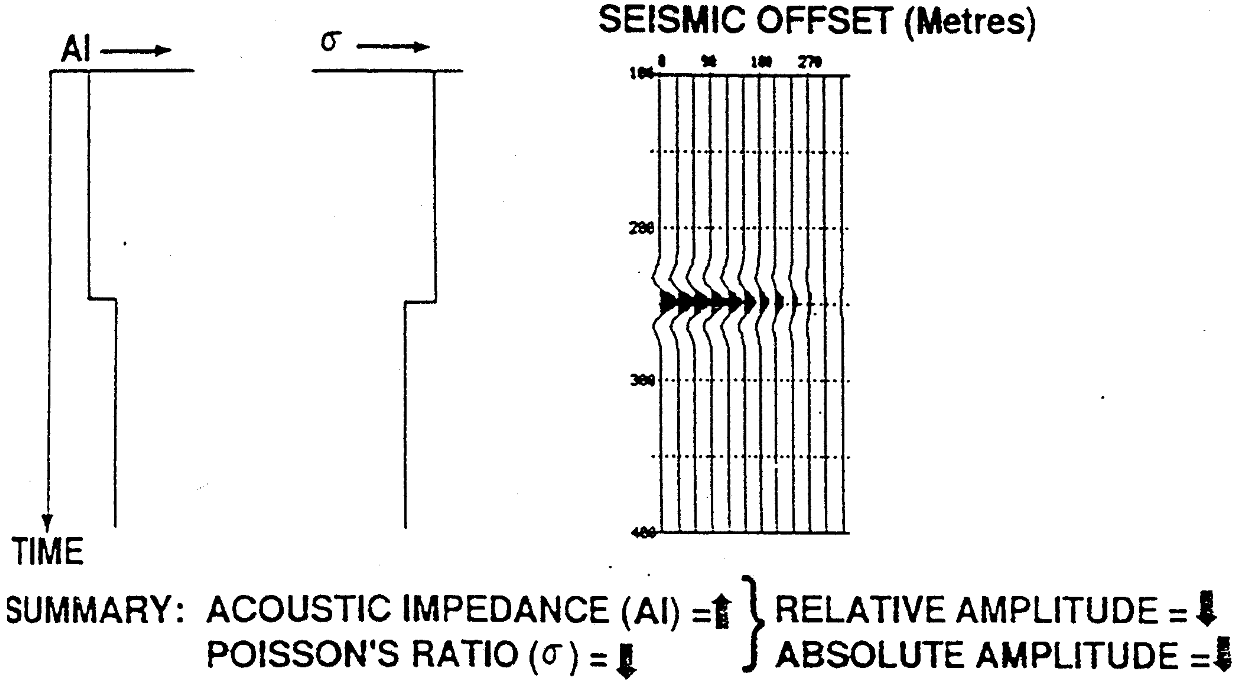
- Higher angles incorrect
- Near the critical angle the approximations are invalid
- Assume small changes in elastic parameters across boundaries so at large impedance contrasts, e.g. gas sands or carbonates, the equations go wrong
- So why do it?
- Most seismic was low incidence angle
- Ease of calculation
- Save computing time when repeating millions of calculations

Intuitive Development of the AVO Response



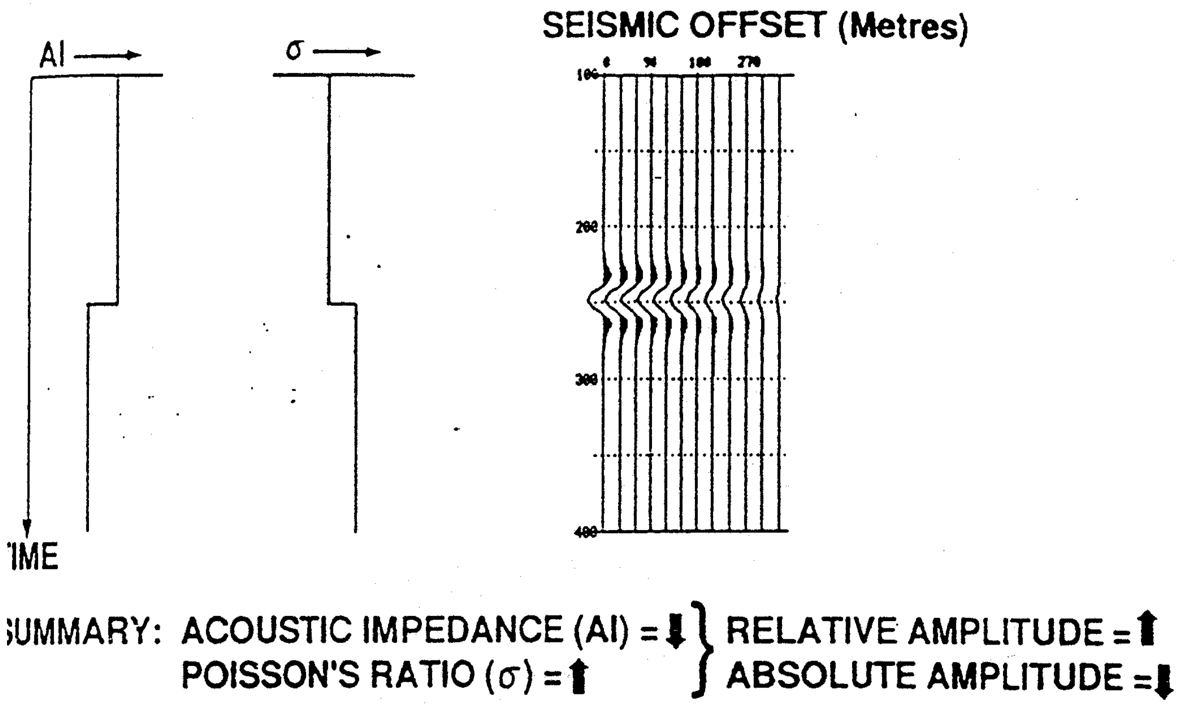
AVO model showing the effect of increasing Poisson's Ratio and AI (Russel, 1998)

Intuitive Development of the AVO Response



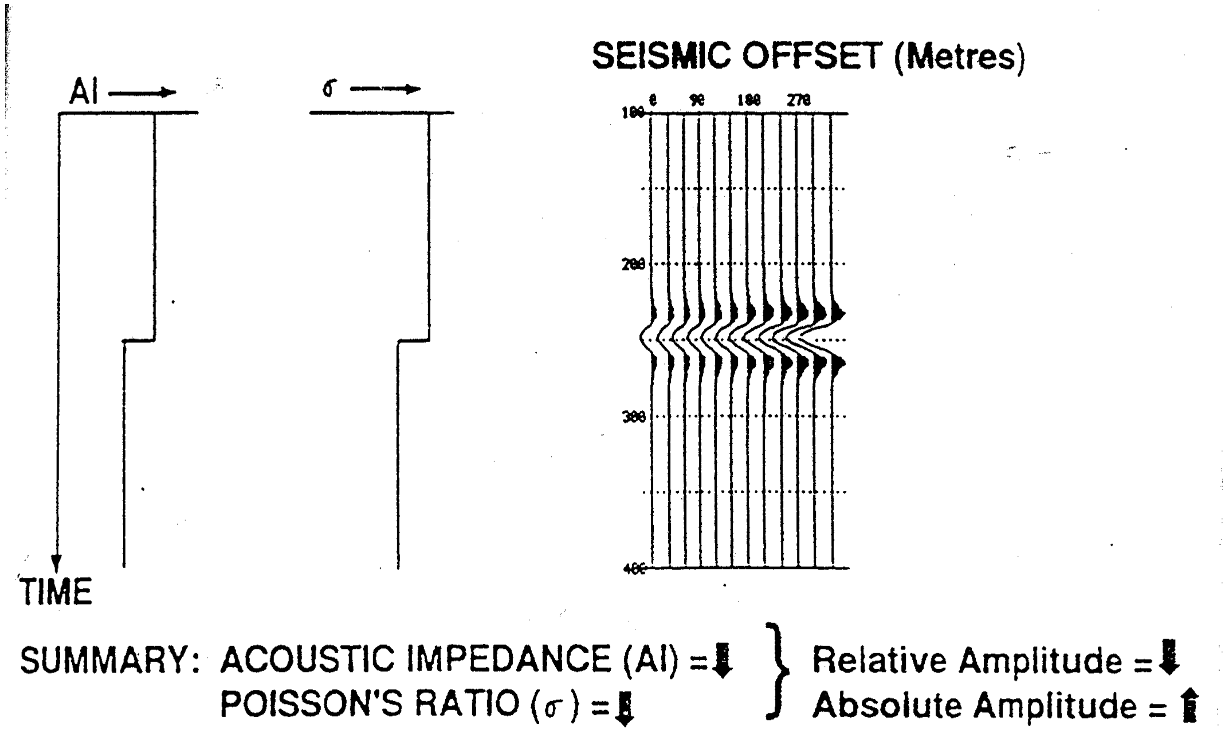
AVO model showing the effect of increasing AI and decreasing Poisson's Ratio (Russel, 1998)

Intuitive Development of the AVO Response



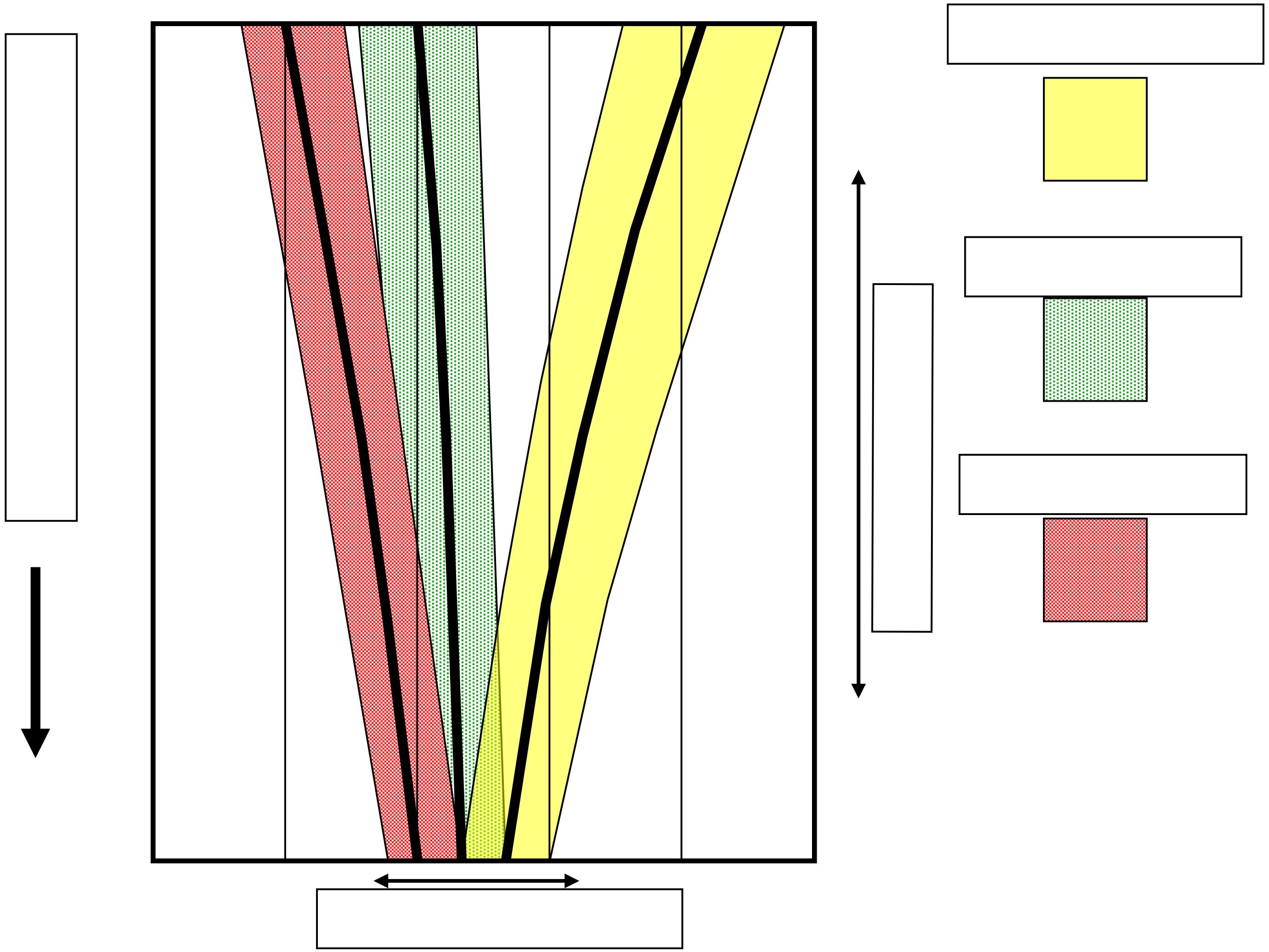
AVO model showing decreasing AI and increasing Poisson's Ratio (Russel, 1998)

Intuitive Development of the AVO Response

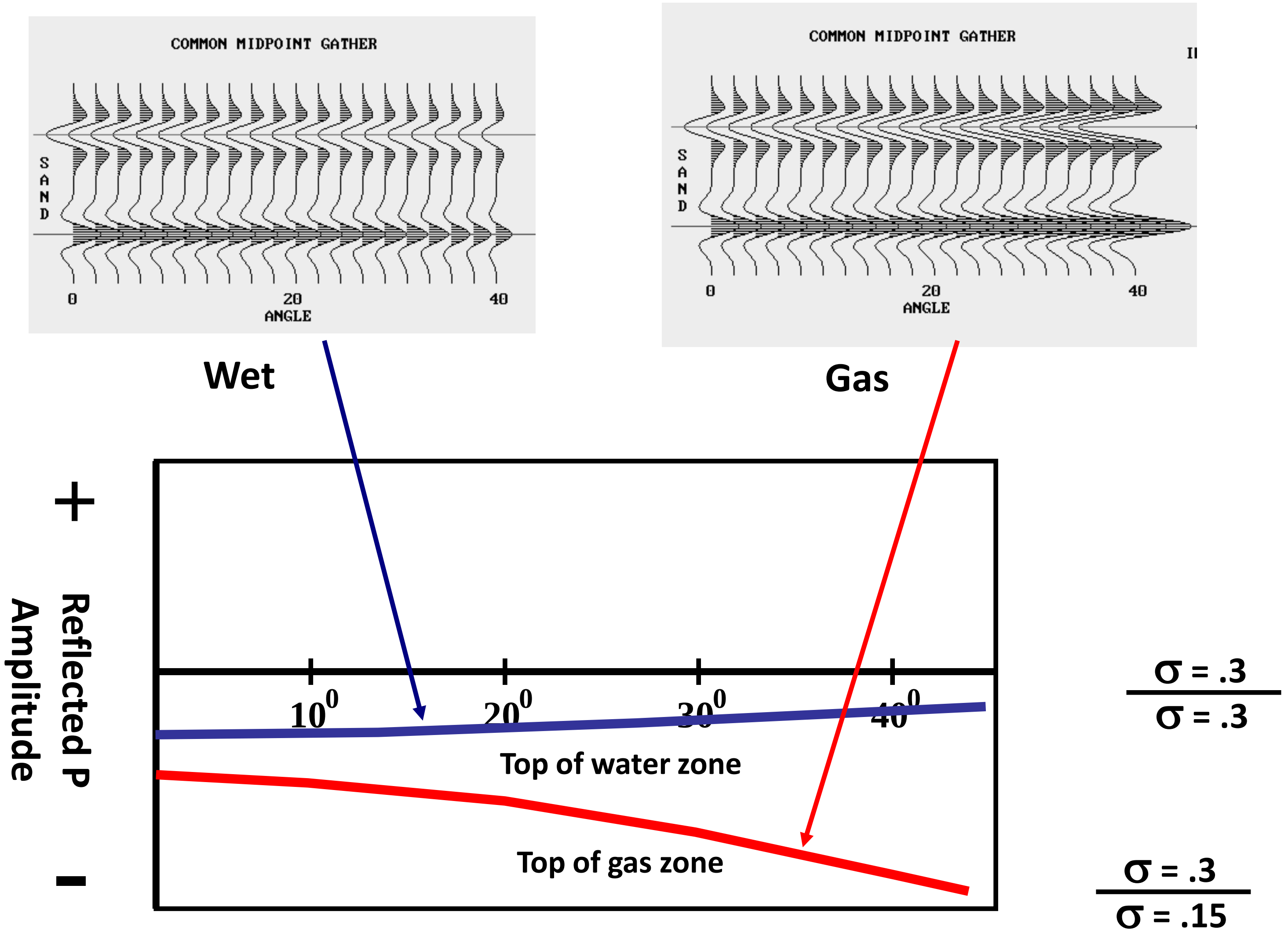


AVO model showing decreasing AI and Poisson's Ratio (Russel, 1998)

Intuitive Development of the AVO Response



Intuitive Development of the AVO Response



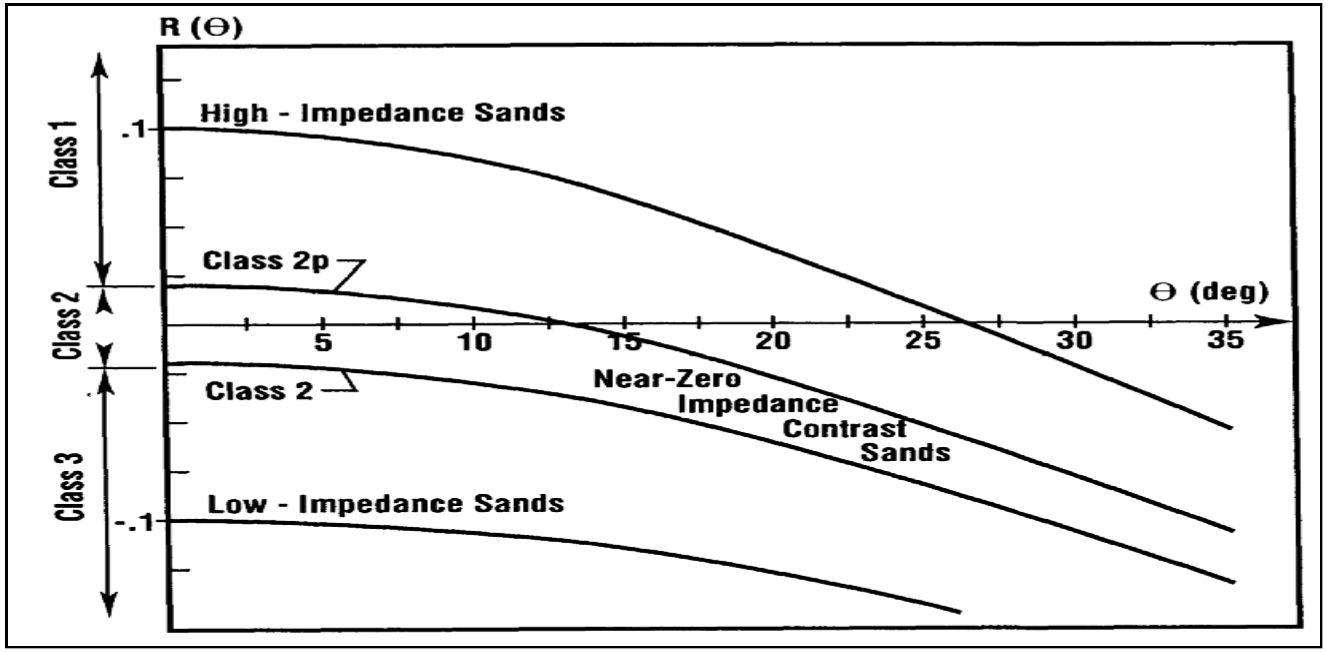
AVO Classification Scheme

Rutherford and Williams (1989) proposed that there were three classes of gas-sandstone reservoirs, i.e :

Class 1 : high impedance gas-sands

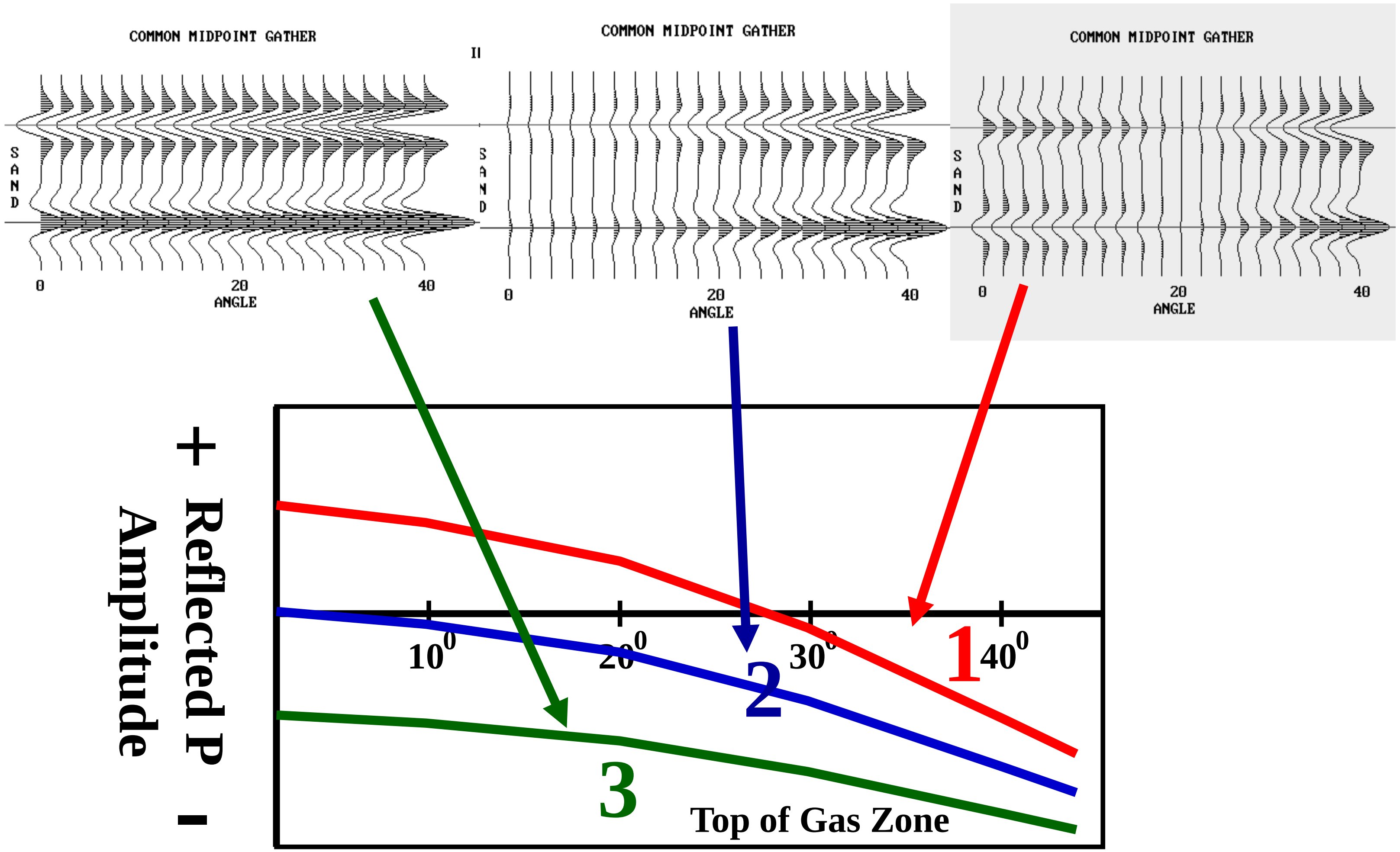
Class 2 : near zero impedance contrast gas-sands

Class 3 : low impedance gas-sands

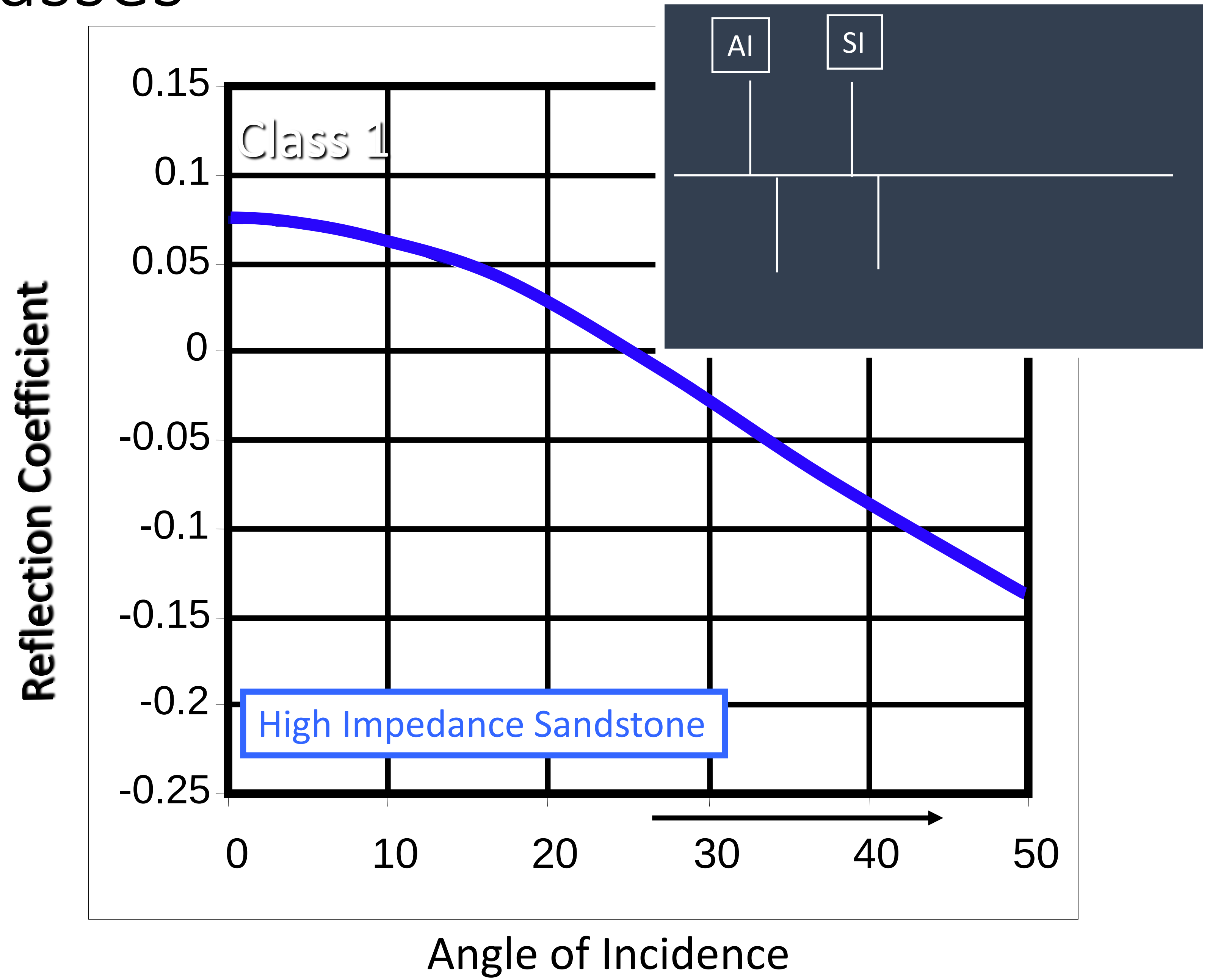


AVO Classification Scheme

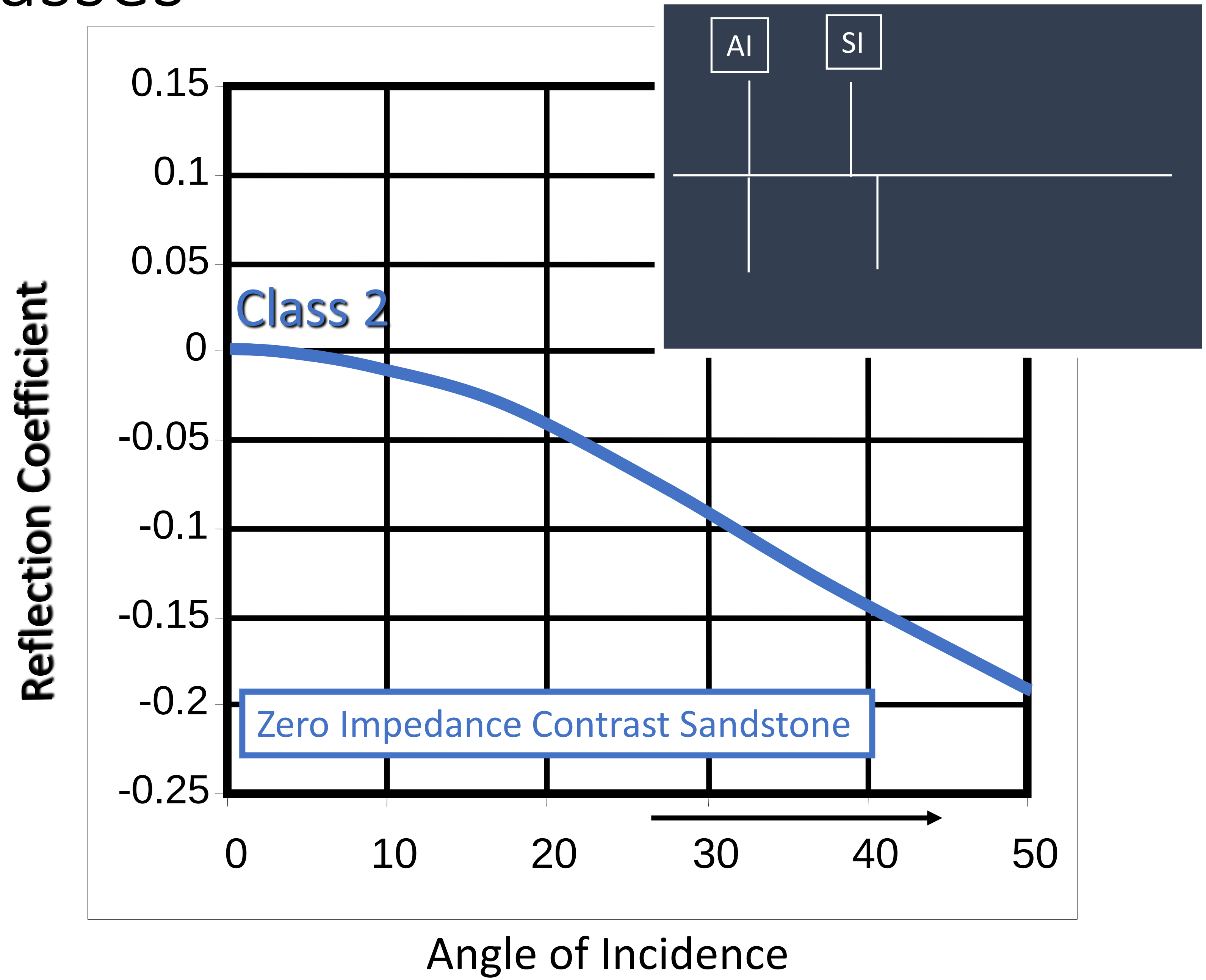
Rutherford and Williams



AVO Classes

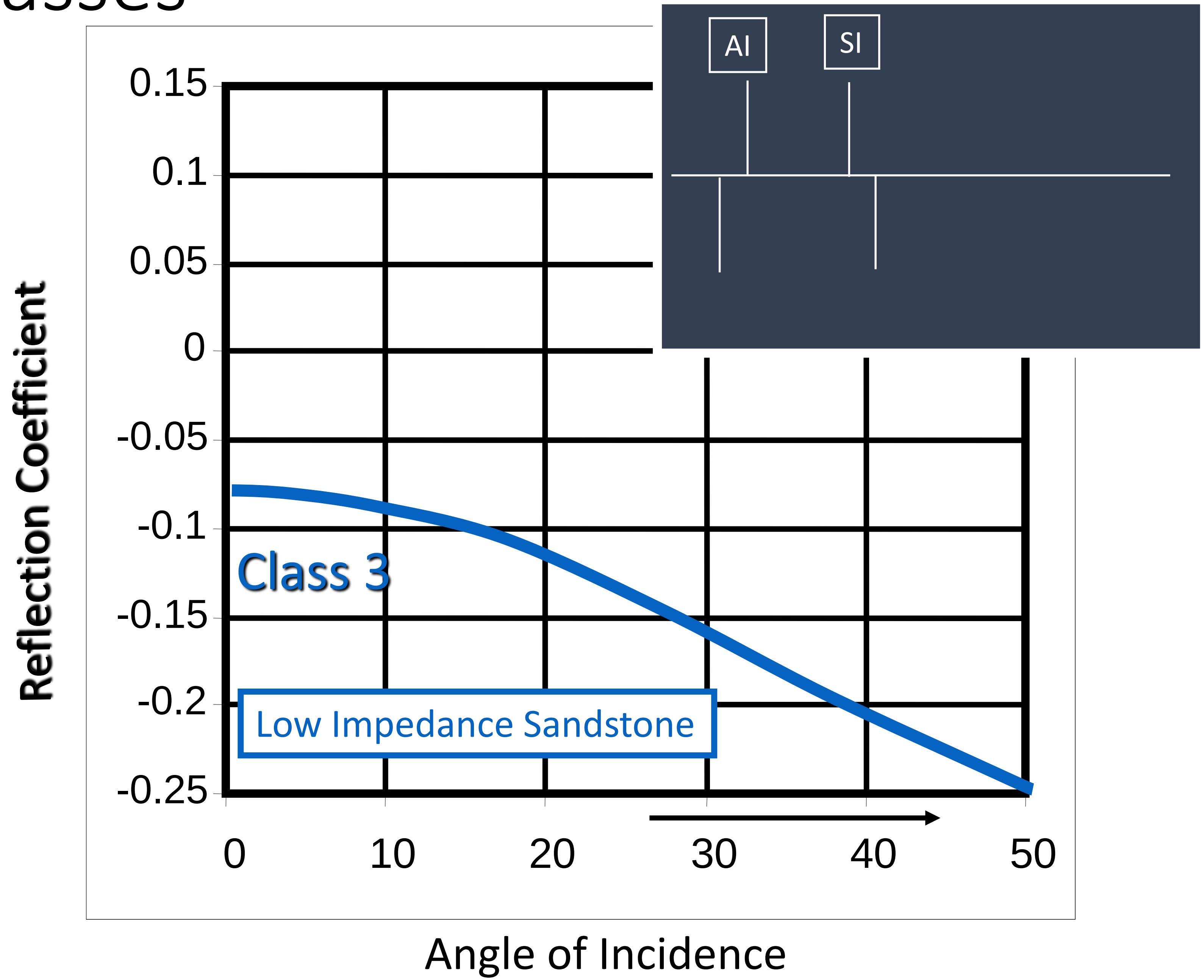

AI
SI
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
Class 1
Reflection Coefficient
High Impedance Sandstone
0
10
20
30
40
50
Angle of Incidence

AVO Classes



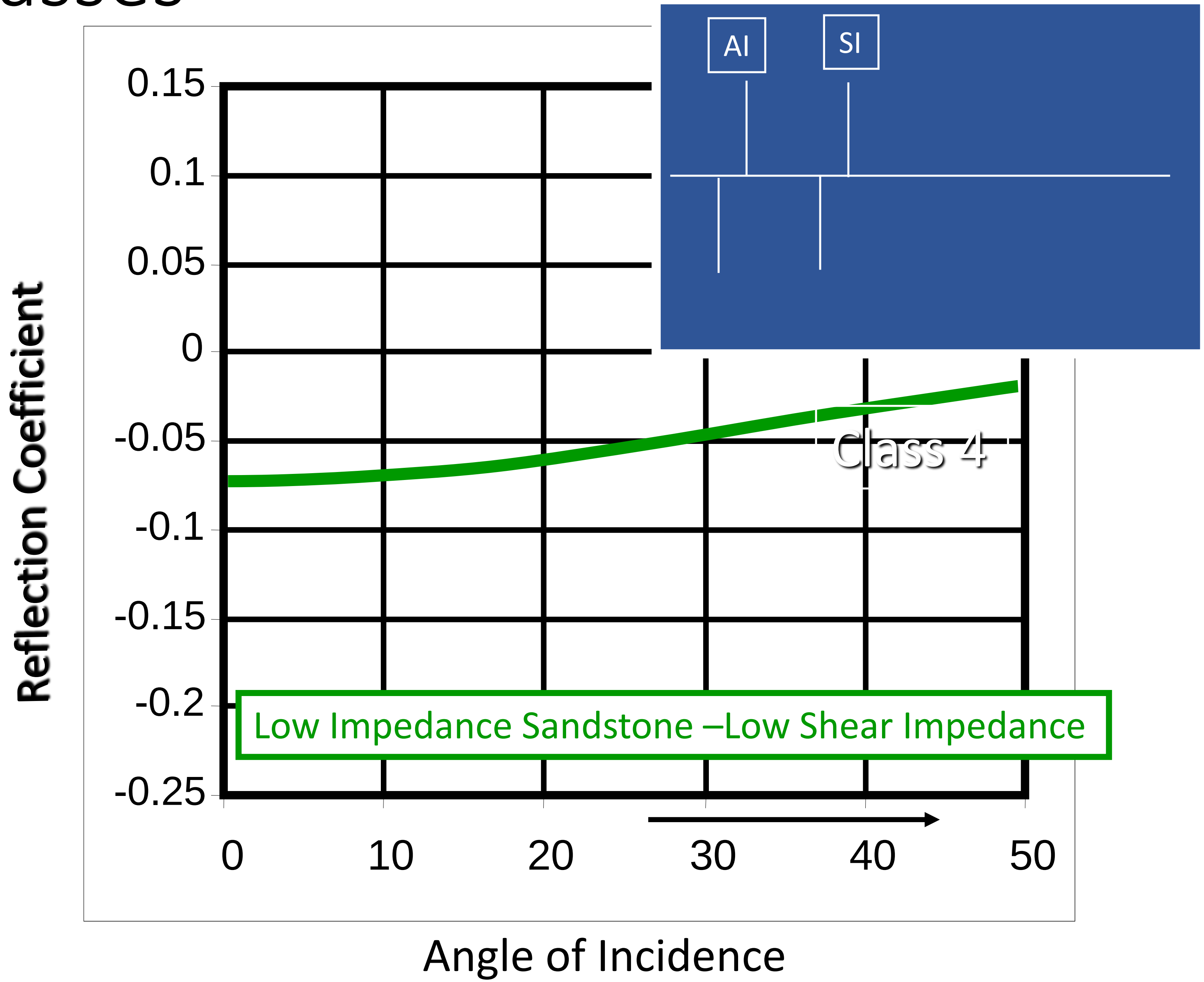
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
10
20
30
40
50
Reflection Coefficient
Class 2
Zero Impedance Contrast Sandstone
Angle of Incidence
AI
SI

AVO Classes



AI
SI
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
Reflection Coefficient
Class 3
Low Impedance Sandstone
0
10
20
30
40
50
Angle of Incidence

AVO Classes


AI
SI
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
Reflection Coefficient
Class 4
Low Impedance Sandstone –Low Shear Impedance
0
10
20
30
40
50
Angle of Incidence

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Rutherford and Williams

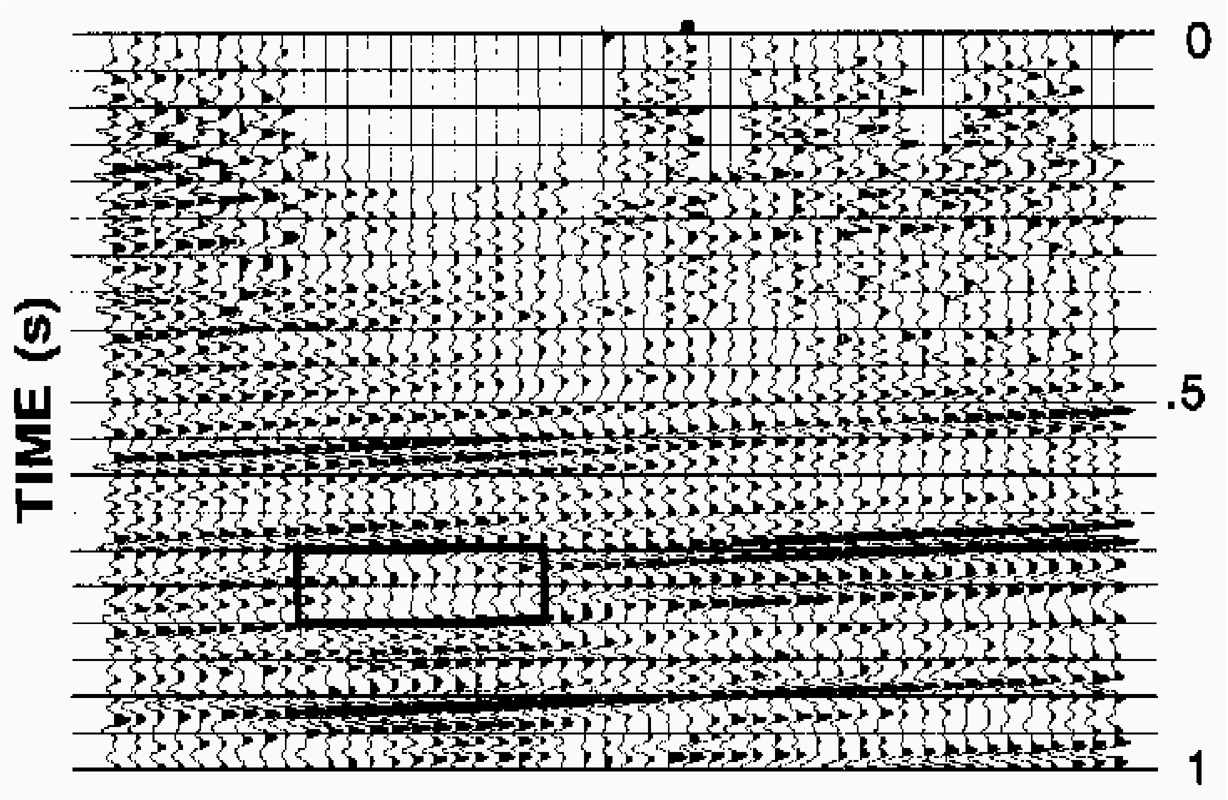
Class 1 Anomaly: High Acoustic Impedance Sands

Sandstone Class 1 has relatively high impedance than its seal layer, which usually is shale. Interface between shale and this sandstone will result relative high positive coefficient reflection (R_0) and a negative coefficient at the base.

Usually this sandstone is found in coastal exploration area. This sandstone is a mature sandstone which have moderately to highly compacted.

AVO Classification Scheme

Rutherford and Williams



Class 1 anomaly: A stacked seismic section on Hartshorn field. The productive interval corresponds to the dim out phenomena which highlighted in the figure. The dim out is caused by a change in polarity with offset of the Hartshorn reflection (Rutherford and Williams, 1989)

AVO Classification Scheme

Rutherford and Williams

Class 2 Anomaly: Near-Zero Acoustic Impedance Contrast Sands

Sandstone class 2 has almost the same Acoustic Impedance as the seal rock. *This sandstone is a compacted and moderately consolidated sandstone.*

Gradient of sandstone class 2 usually has big magnitude, but generally it’s smaller than the magnitude of sandstone class 1.

Reflectivity of sandstone class 2 on small offset is zero. This is often blurred due to the presence of noise on our data seismic.

AVO Classification Scheme

Rutherford and Williams

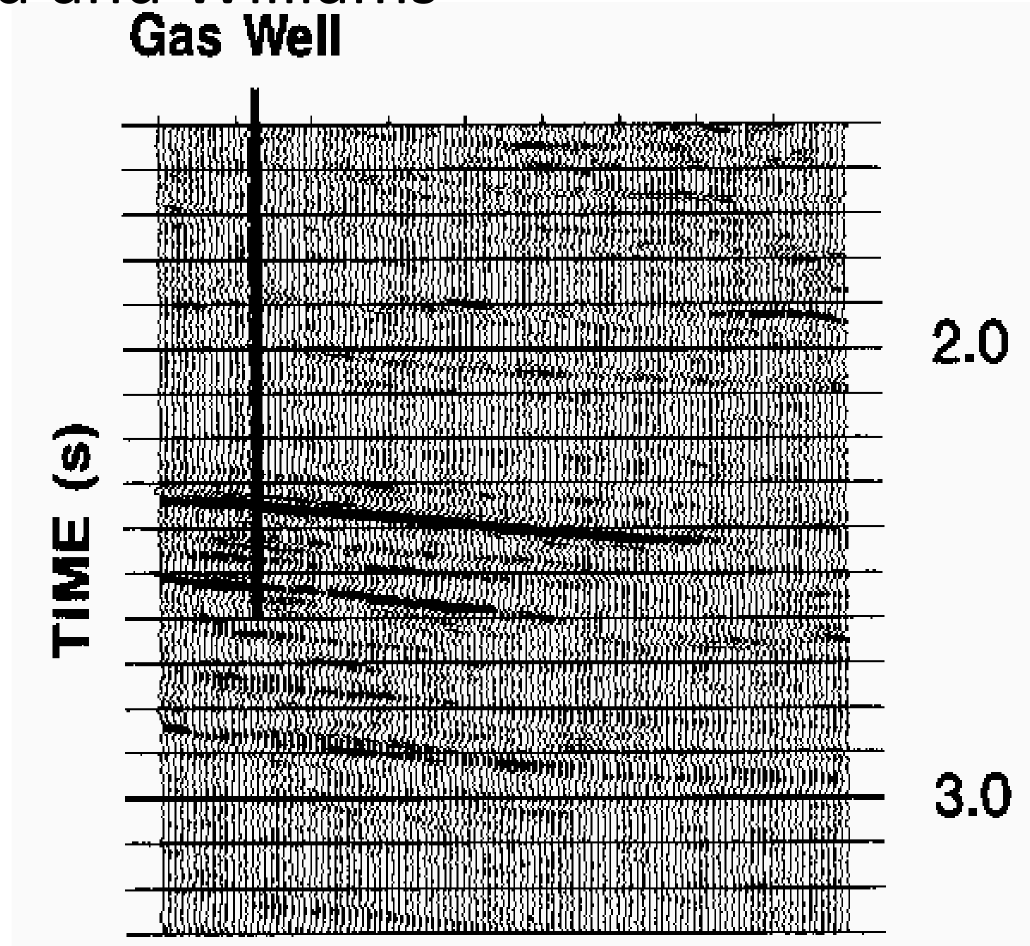
Class 3 Anomaly: Low Acoustic Impedance Sands

Sandstone class 3 has lower acoustic impedance than the seal rock. *Usually this sandstone is the less compacted and unconsolidated sandstone.*

On seismic stack data, sandstone class 3 has big amplitude anomaly and reflectivity in the whole offset.

Usually, the gradient is significant enough but it has lower magnitude than the sandstone class 1 and 2 during the RC's normal incidence angle is always negative.

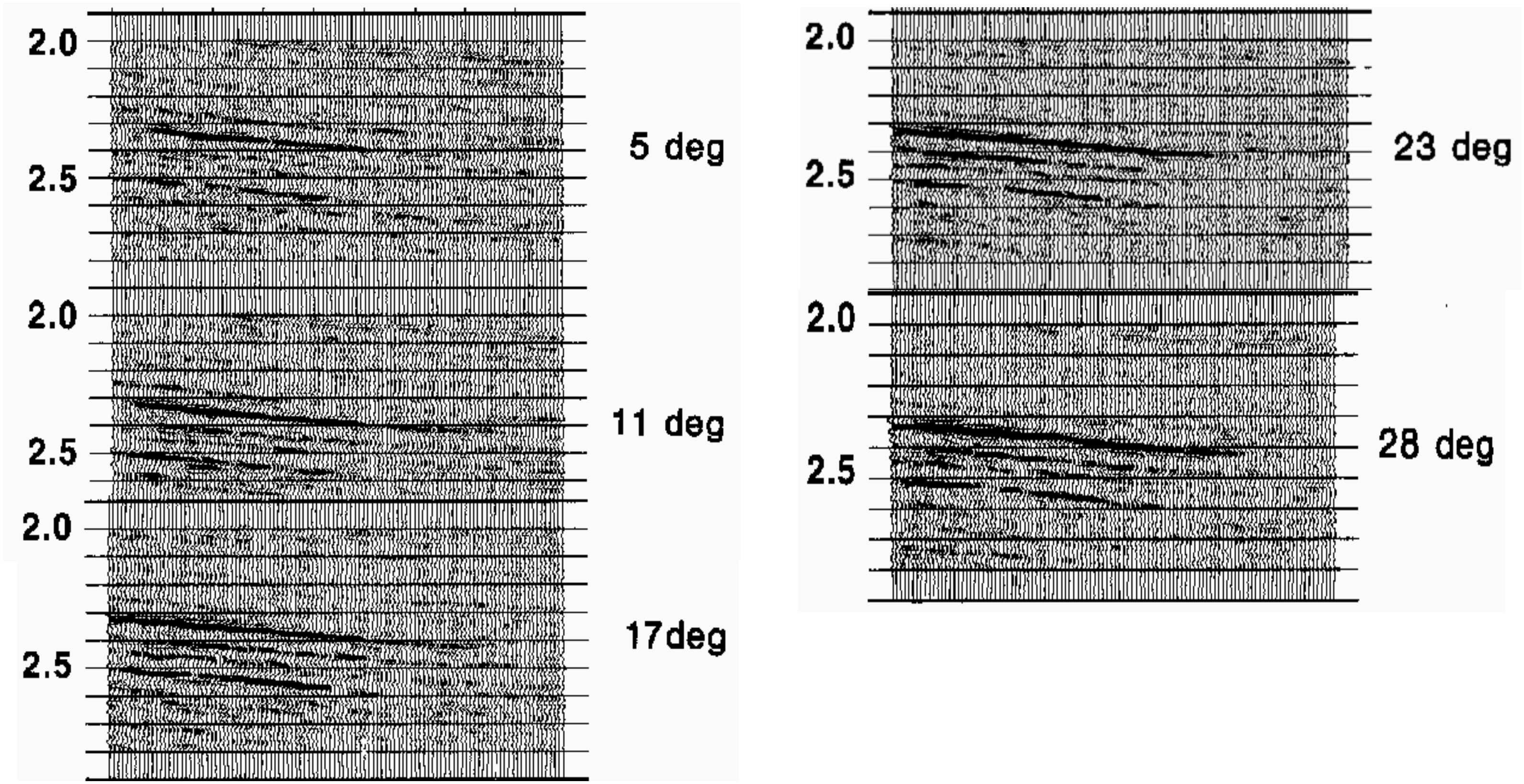
Rutherford and Williams



Class 3 Anomaly: Migrated, stacked seismic section on Pliocene gas-sand in High Island Gulf of Mexico. The reflector of interest is between 2.3 s and 2.5 s (Rutherford & Williams, 1989)

AVO Classification Scheme

Rutherford and Williams



Class 3 Anomaly: Panel display of constant reflection angle sections corresponding to the stacked data on previous slide. Each panel displays 2.0 s to 2,8 s of data
(Rutherford & Williams, 1989)

AVO Classification Scheme

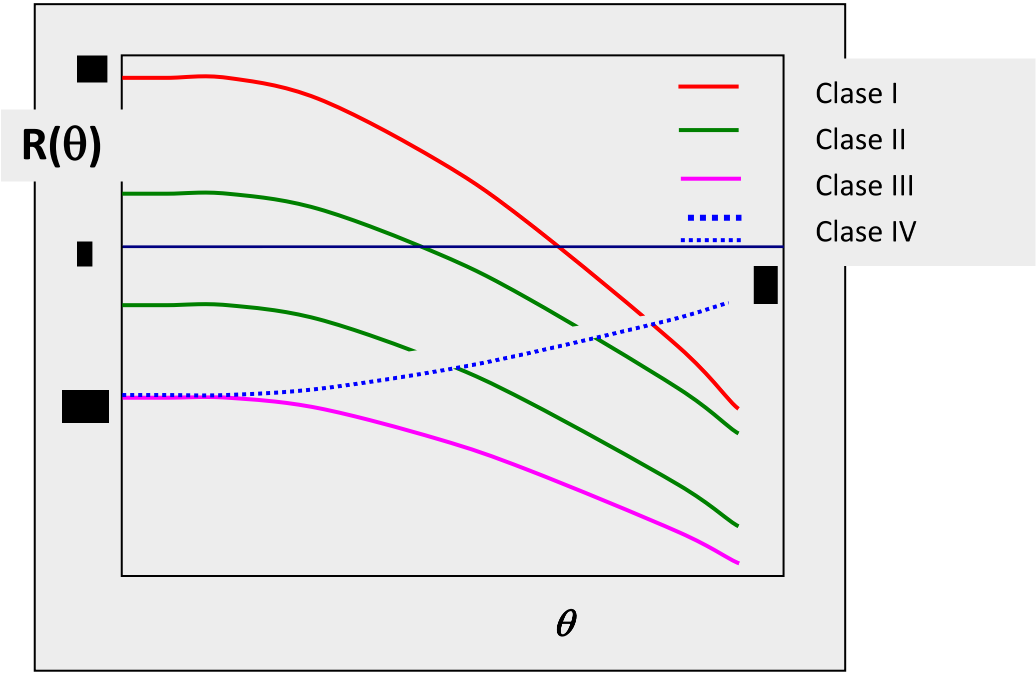
Rutherford and Williams

Class 4 Anomaly:

The fourth class of gas-sandstone is the anomaly with the reflection coefficient becoming positive along as offset increases, but the magnitude decreased as the offset increases.

Sandstone class 4 often emerged when the porous sandstone, which is overlain by lithology which have higher shear wave velocity, such as hard shale (e.g : siliceous or calcareous), siltstone, tightly cemented sand, or carbonate.

- **Class I:** Sands of greater impedance than the surrouding media where a **dim-out** is present.
- **Class II:** Sands of similar impedance than the surrounding media. **Phase reversal**
- **Class III:** Sands of lower impedance than the surrounding media. **Bright spot**
- **Class IV:** <u>Low impedance</u> reservoirs where magnitude **<u>decreases</u> with offset** (a sort of dim-out)



Possible Causes of Amplitude Anomalies

- Hydrocarbon
- Coal
- Improper amplitude balance
- Fizz water
- Overpressure
- Lithology change
- Improper NMO
- Improper migration
- Tuning
- Frequency imbalance (between near and far stack)
- Multiples
- Noise
- Edge effects (refraction at low angles< 35^{o})

AVO Analysis - **A Word of Caution**

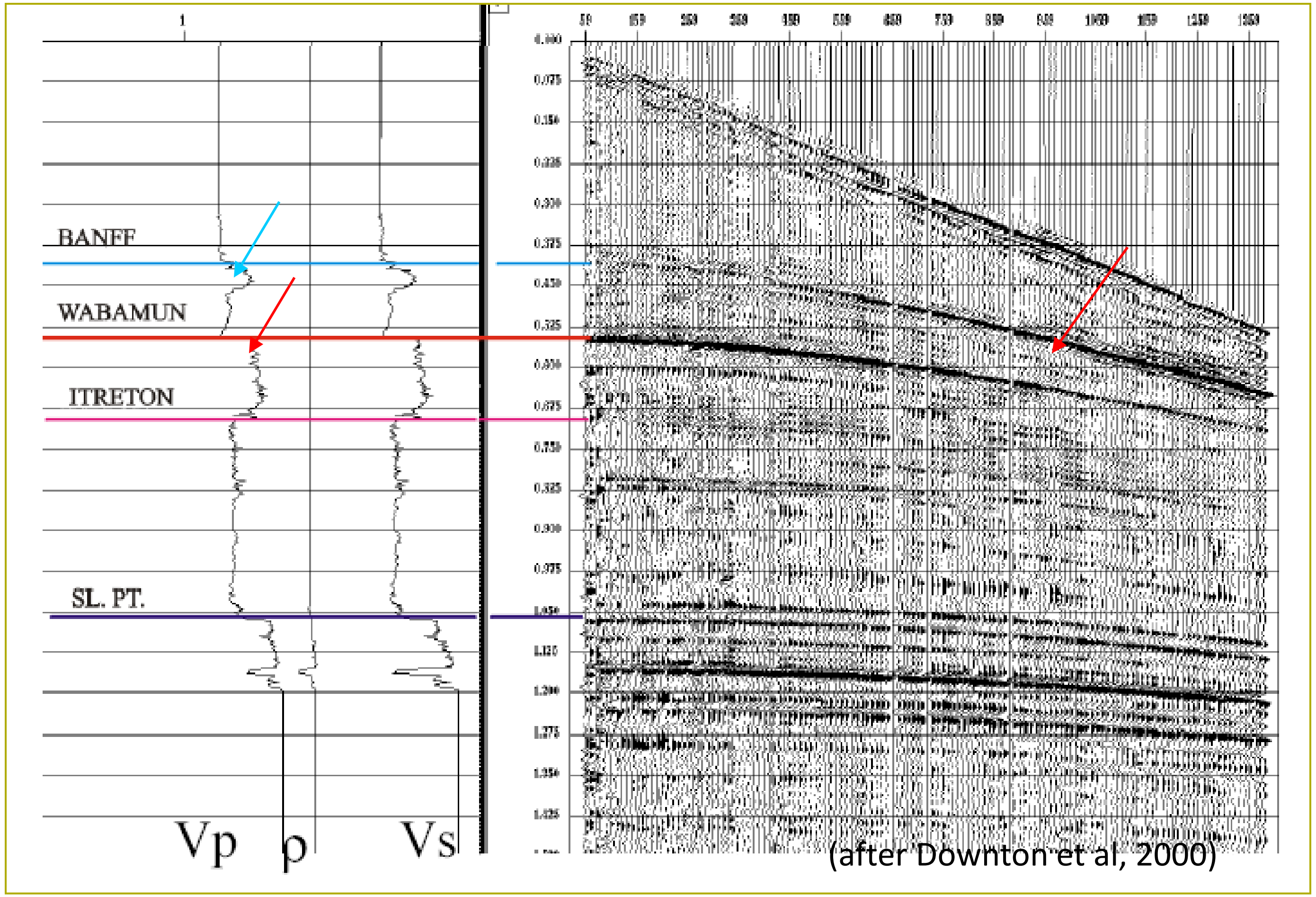
AVO attributes are quite useful in interpretation:

- Increase reservoir evaluation
- Helps to understand the relationship between rock and fluids natures
- Play role in hydrocarbon delineation

However, there are a number of pitfalls:

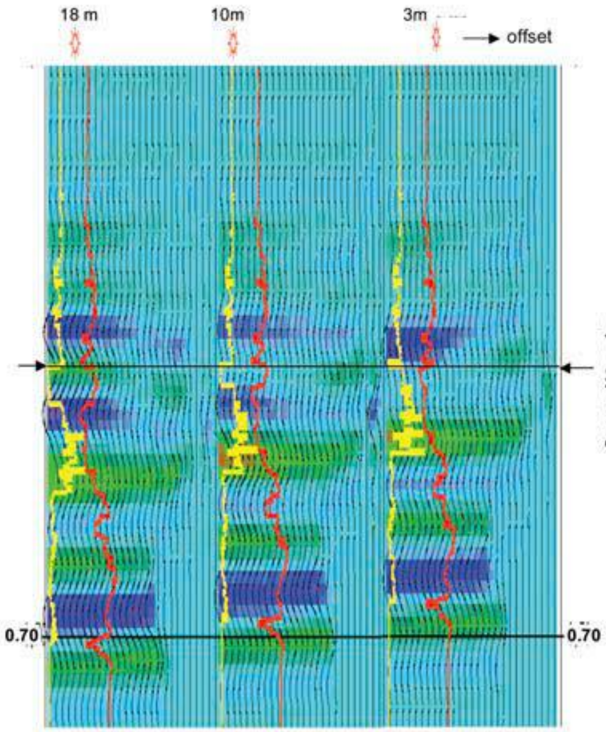
- Transmission losses can change as a function of offset due to large velocity contrast above target
- Inadequate AVO data processing: often processing done to create an optimal stacked section will make it unsuitable for AVO analysis (trace scaling/balancing on prestack gathers)

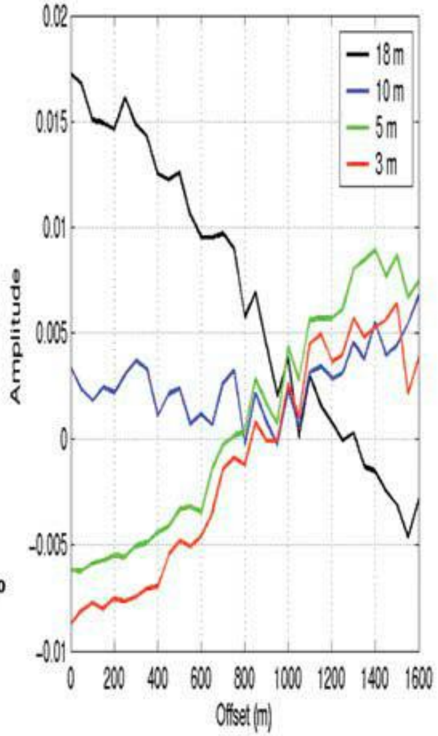
AVO Analysis - A Word of Caution



The Wabanum and Ireton reflections show amplitude decrease with offset. However, offset dependant transmission losses occur above on Banff and Wabanum interfaces where high velocity contrast take place.

Effect of tuning on AVO





Li et al., 2007