

Chapter 1

Rock Physics

Rock-physics | Why

Rock physics uses the principles of physics to establish relationships between fundamental rock and fluid properties and seismically observable quantities such as the speed with which seismic waves travel, and why and how they reflect

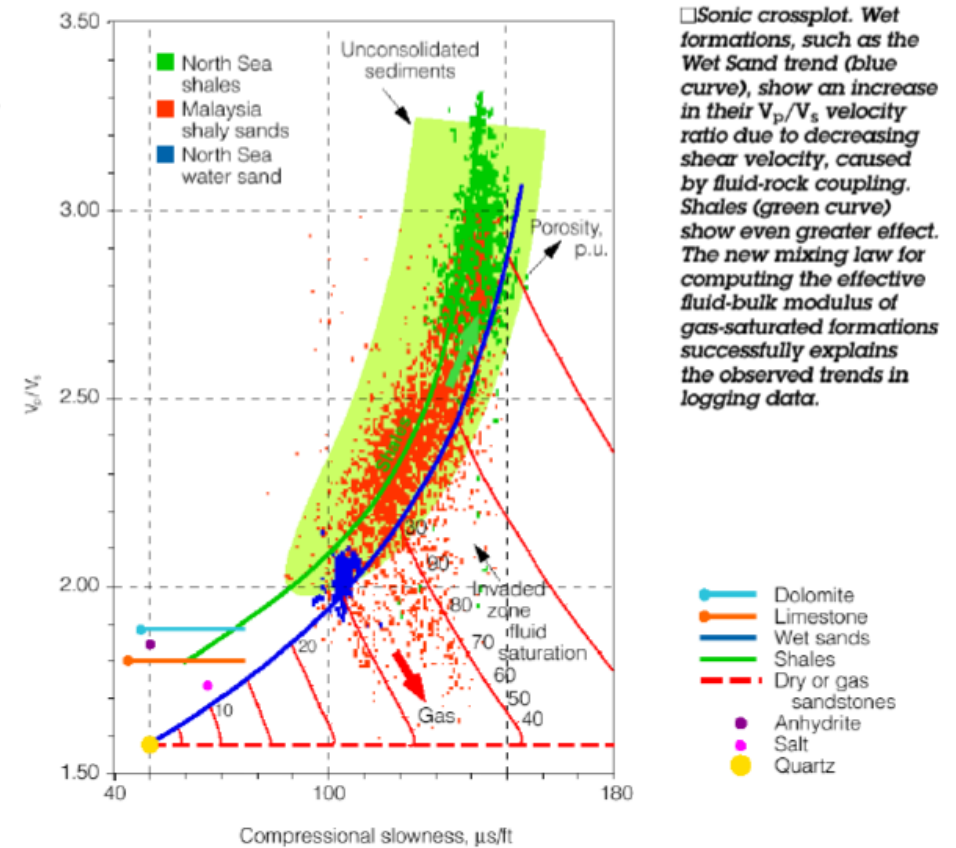


Rock-physics | Why

The elastic properties are the only parameters, which are obtained directly in seismic measurements. Velocity of propagation depends on the physical elastic properties of the rock it travelled through

It is therefore fundamental e.g. for exploration and production purposes that the influence of porosity, pore fill and pressure on the elastic properties is adequately described

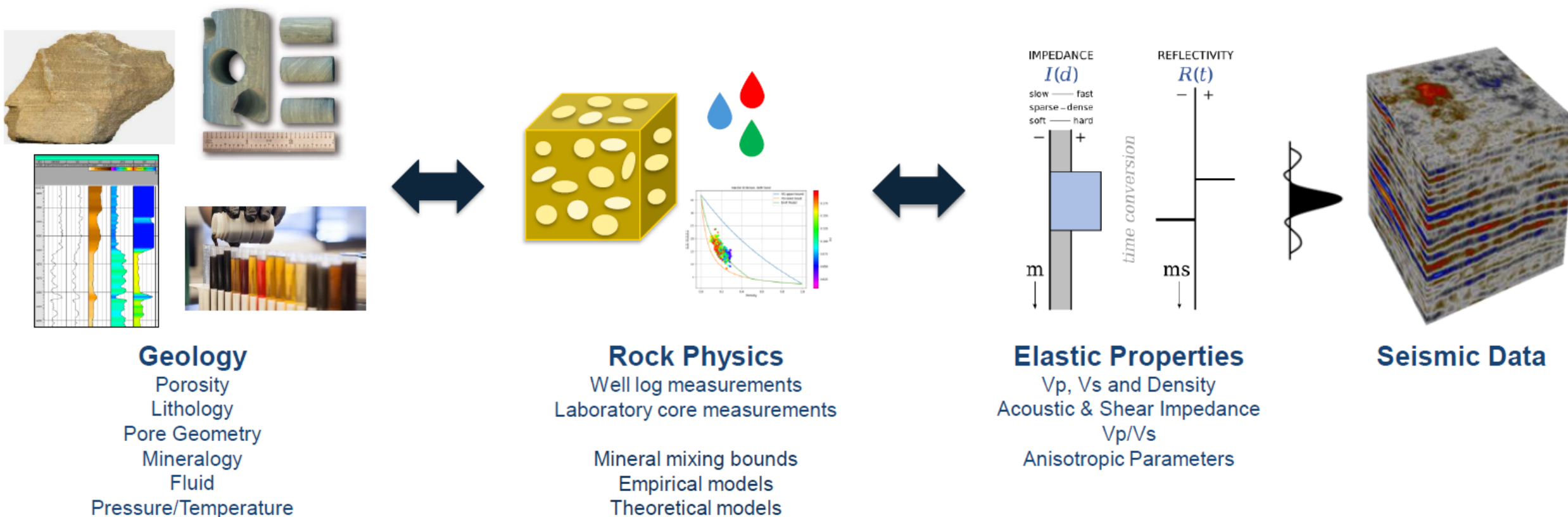
Templates showing empirical relationships used to characterize the observed trends in logging data and potentially in calibrated seismic attributes



Shear Sonic Interpretation in Gas-Bearing Sands (SPE 30595), Brie, Pampuri, 1995

Rock-physics | What?

Rock physics provides the quantitative link between the elastic **rock** properties that affect seismic response and the geological properties that we are trying to predict. (Hamlyn 2016)



Differences between rock physics and Petrophysics

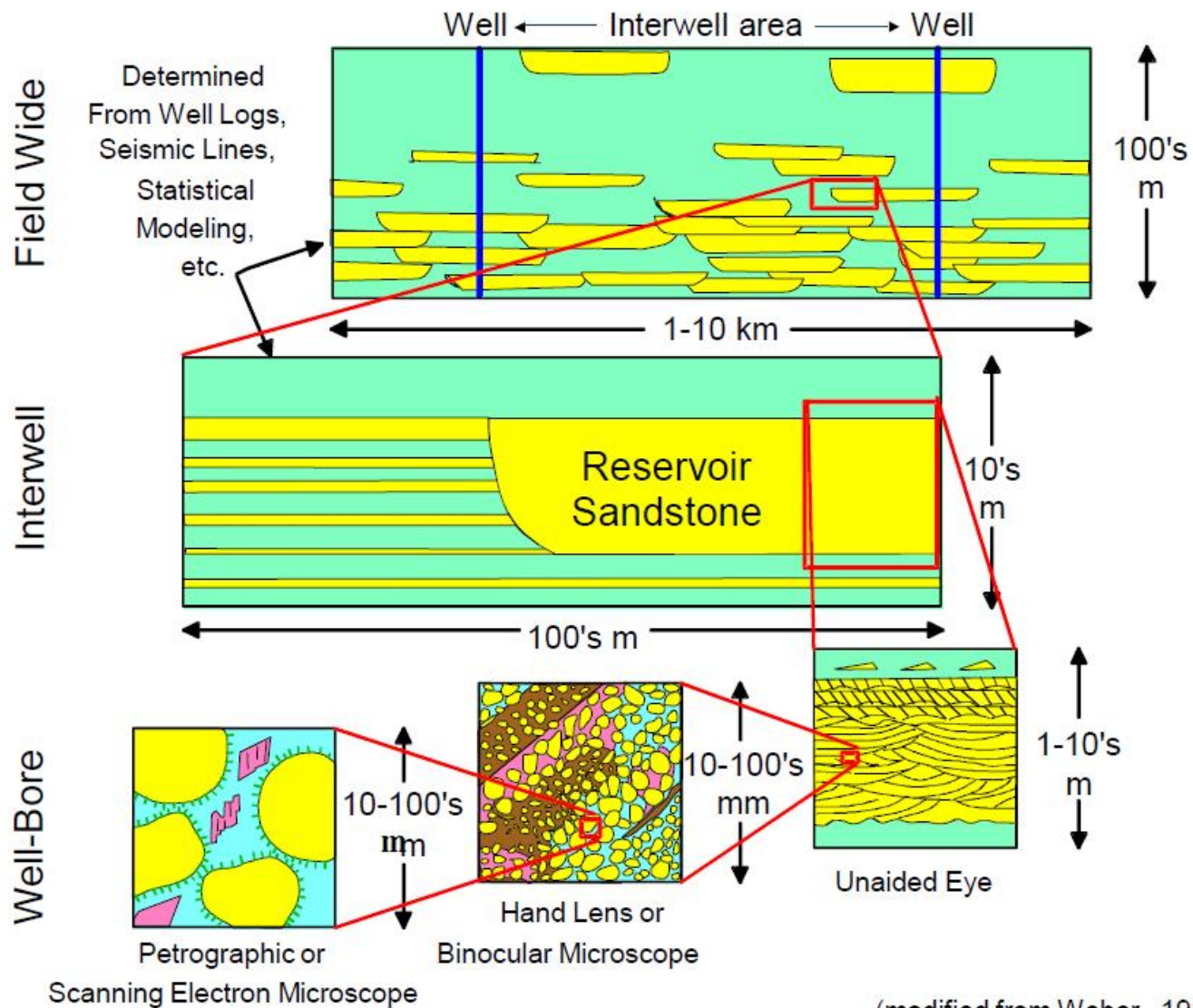
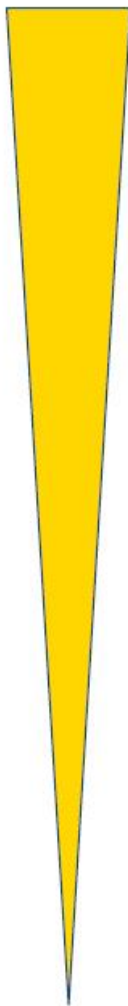
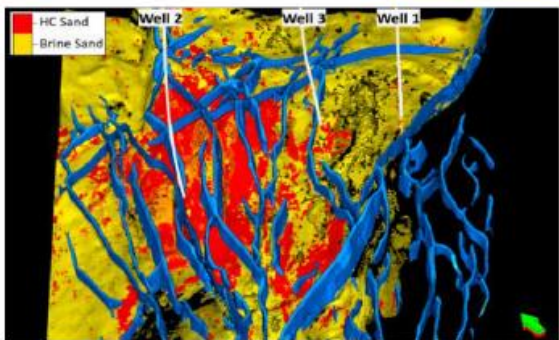
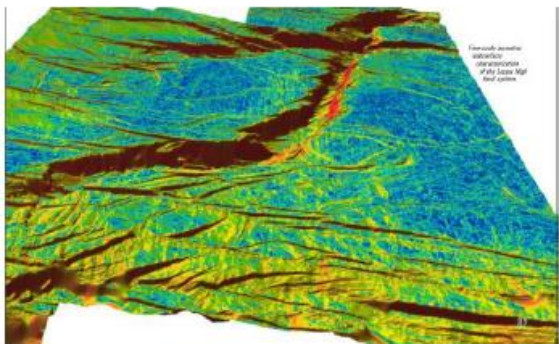
Rock Physics

- Rock physics is concerned with the description of the physical properties of reservoir rocks and their **effects on seismic wave propagation**.
- Uses open hole logs (density, sonic, Gamma Ray, Neutron, etc) and petro- acoustic logs (porosity, saturation, litho-logs, velocities, etc) and seismic attributes to describe reservoir rocks through **theoretical or empirical relationships**.
- Rock physics **links** the geosciences together – geophysics, geology, Petrophysics etc

Petrophysics

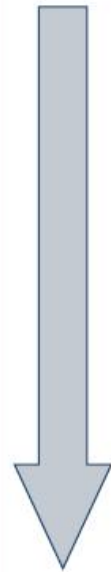
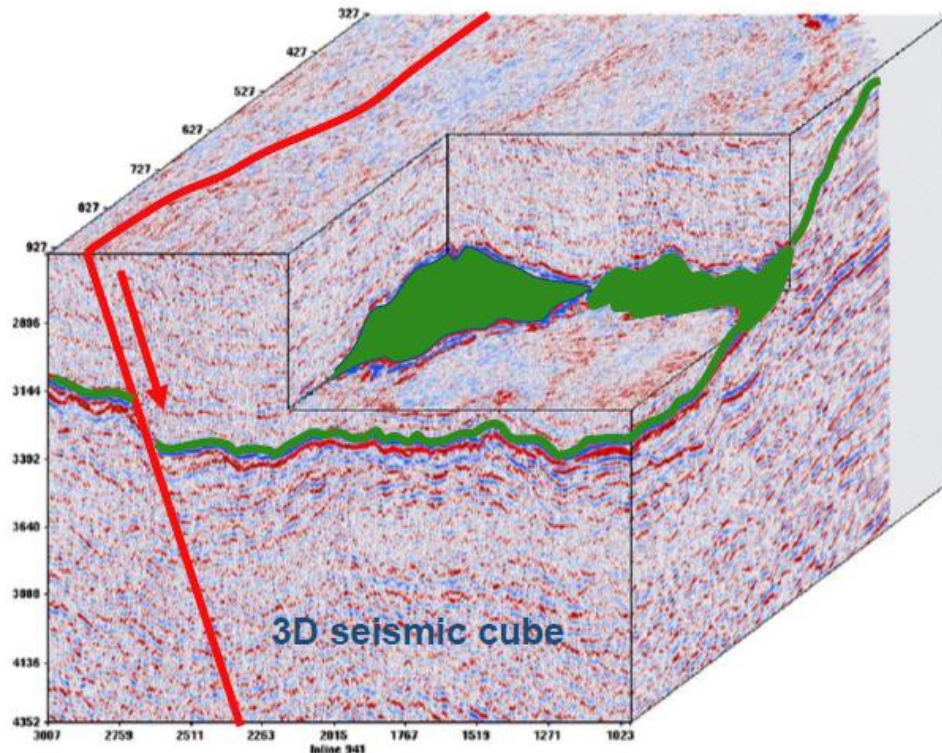
- Petrophysics is the study of rock properties and their **interactions with fluids** e.g. Sw from resistivity logs.
- Directly describes reservoir rocks through **direct measurements** of open and petro-acoustic logs as well as core measurements and is not necessarily interested in their theoretical or empirical relationships e.g. porosity and volume of mineral estimation.
- Petrophysics is typically concerned with **reservoir evaluation** through log and core analysis, volumetric analysis e.g. net pay estimation

Rock-physics | Scale



Rock-physics | Seismic reservoir characterization

Can seismic predict some key characteristics and properties?



Scale

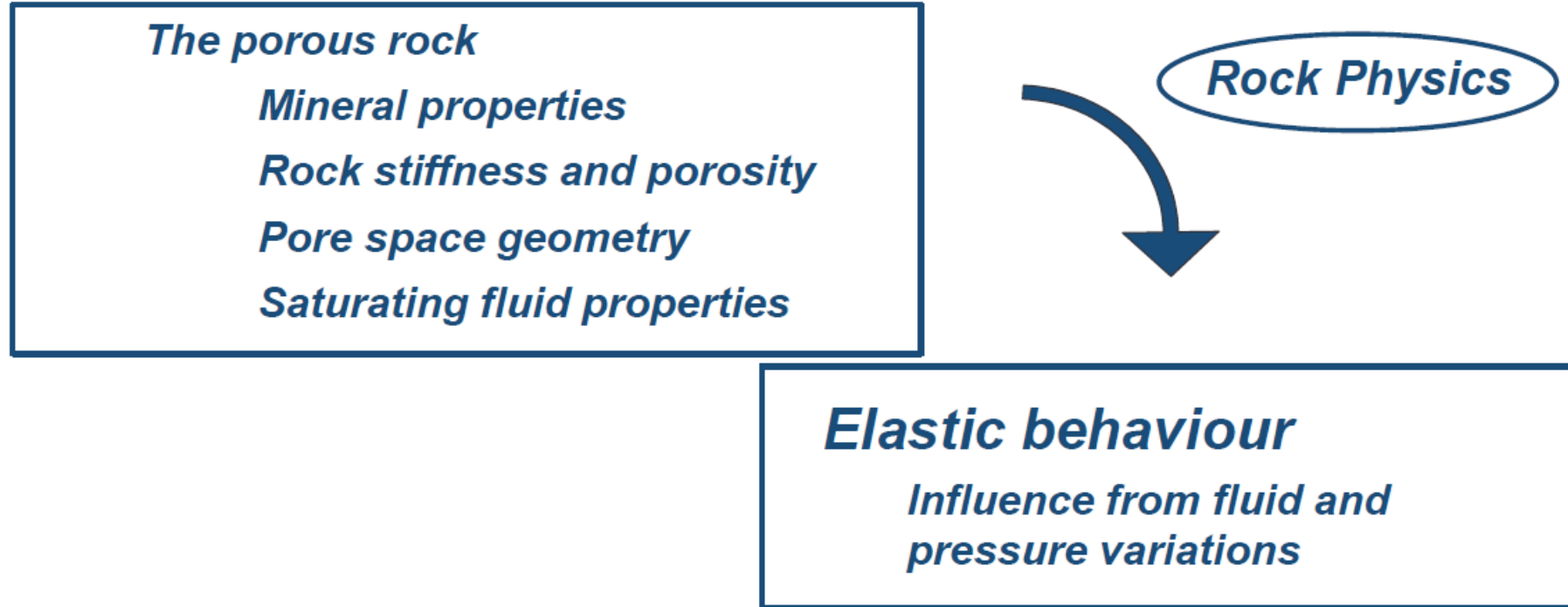
Structure
Stratigraphy
Fractures
Lithology
Pressure
Pore Fluid

Depth
Geological Setting - Origin of Rocks/Fluids
Geological Structure
Geometry – thickness, areal extent, volume, seals
Heterogeneity – Layering, Faults/Fractures, Compartments

Rock Type
Porosity
Fluid Content/Distribution
Pressure Distribution
Changes in pressure/saturation

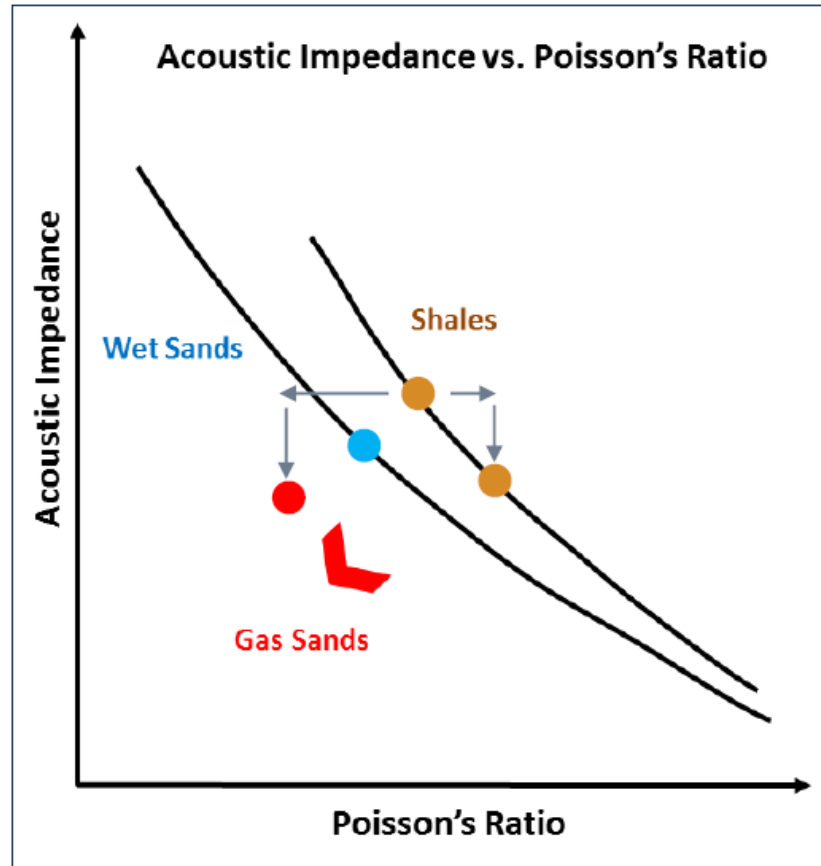
Predict and characterize subsurface reservoirs

Rock-physics | Forward modelling and inversion



Rock-physics | Basic concepts

Lithology and fluid content versus rock properties



Two fundamental rock properties: Acoustic impedance and Poisson's ratio indicating different values for different lithologies

Two reflections: Hard-soft shale and hard shale-gas sand

- Similar decrease in AI but different change in PR

A way to discriminate two similar seismic reflections

Rock-physics | Basic concepts

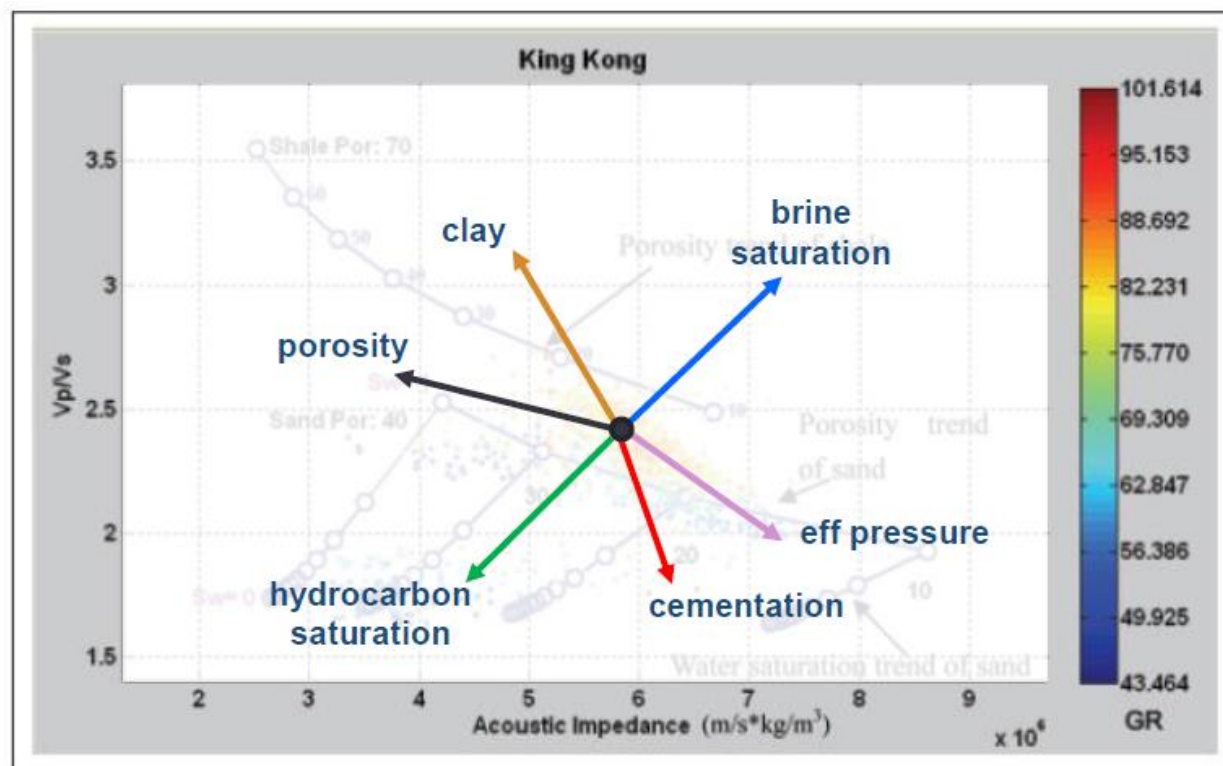
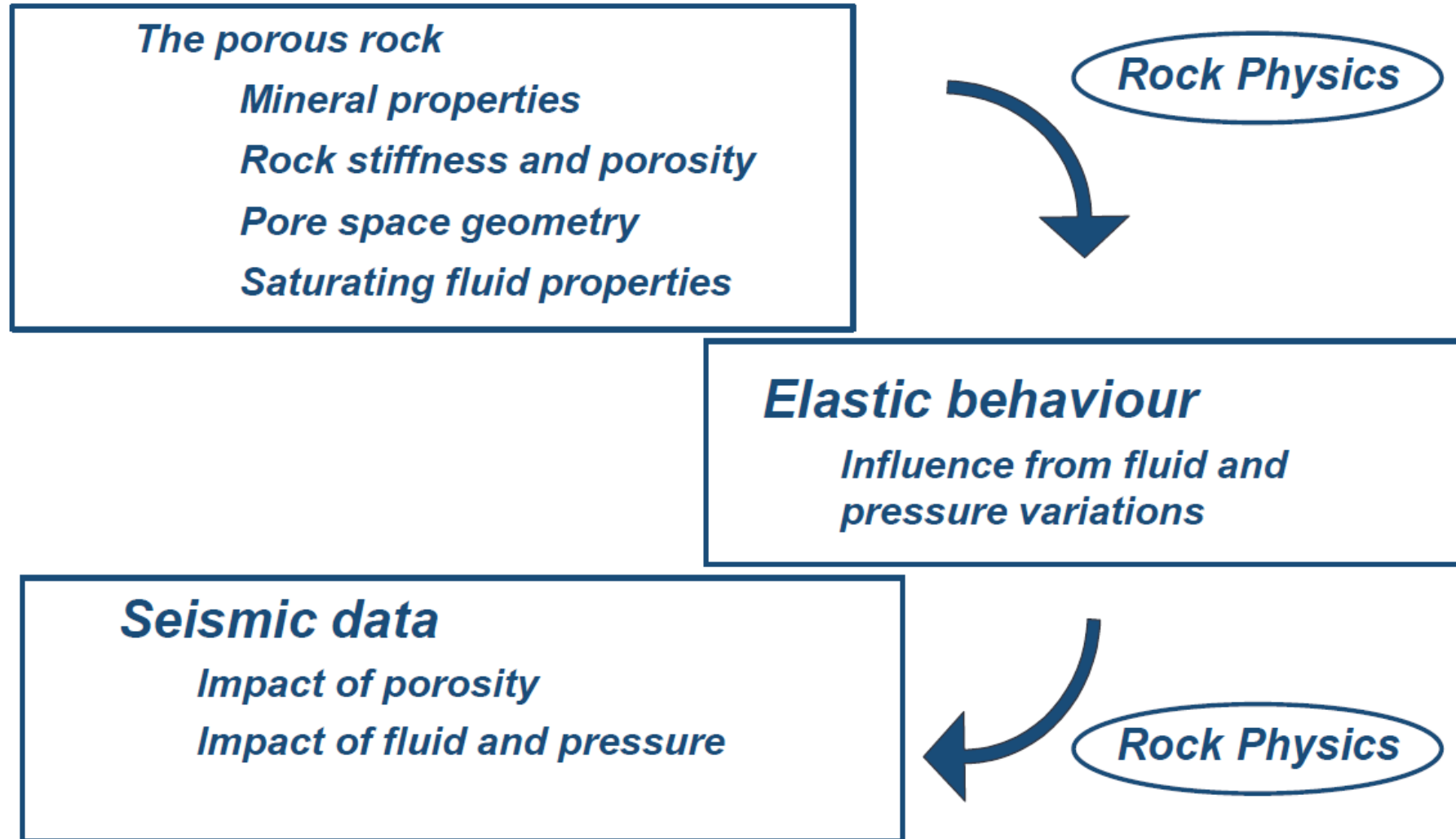


Figure 4. The rock physics template posted in the cross plot of acoustic impedance versus V_p/V_s ratio calculated from well log data at King Kong. The points with low acoustic impedance and V_p/V_s ratio show the high porosity and high gas saturation.

Rock-physics | Forward modelling and inversion

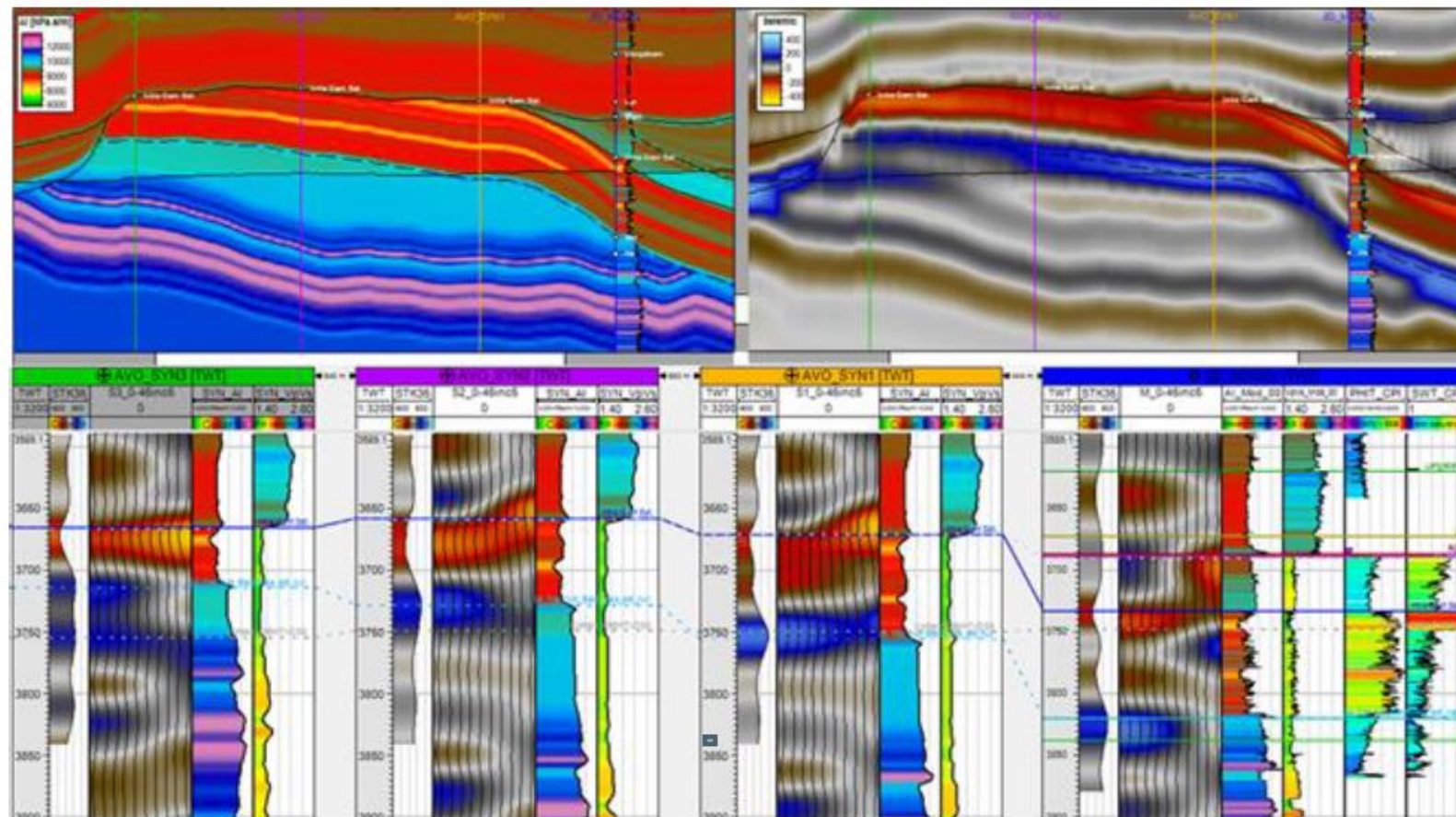


Rock-physics | Basic concepts

Simulate alternative geometrical scenarios to guide the seismic amplitude interpretation away from the well locations.

The rock model sensitivity analysis together with seismic forward modelling incorporating varying sand thickness suggested that sufficiently thick high-porosity sands sitting beneath shales would show a very distinct seismic signature due to the strong contrasts in V_p/V_s ratio, and, in presence of hydrocarbon, moderately strong contrast in acoustic impedance

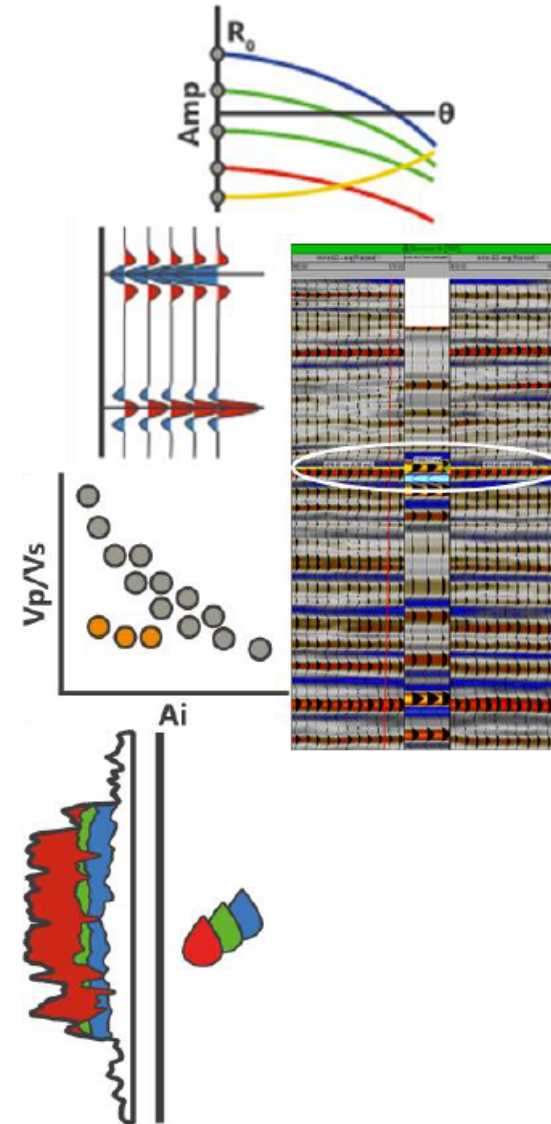
REF: Ø. Tysse, A. Murineddu, R. Bachrach, C. Leone, J.R. Granli, K. Dindane, S. Taggart. Building interpretation confidence with rock physics driven inversion to mature Iris and Hades discoveries ,2nd EAGE Seismic Inversion Conference, Expanded Abstracts.



Upper row, Acoustic Impedance model and resulting synthetic seismic stack (5-30 degrees), and (lower row) synthetic seismic stack (5-30 degrees) and CMP gather (0-45 degrees) responses at 3 representative locations up-dip of the discovery well

Application of rock-physics in exploration and production

- Well log analysis: conditioning and quality control
- Seismic forward modelling
 - Synthetic seismic
 - Interpretation of Amplitude Variation with Offset (AVO)
- Seismic-well tie
- Inversion priors
- Lithology and fluid prediction based on seismic inversion
- Joint porosity-saturation estimation
- Net pay estimation
- Integration with geomechanics
 - Pore pressure modelling/inversion
 - 1D and 3D MEM
- Fracture modelling/prediction
- Time lapse monitoring and calibration
 - Feasibility studies
 - Inversion to changes saturation/pressure (sim2seis)

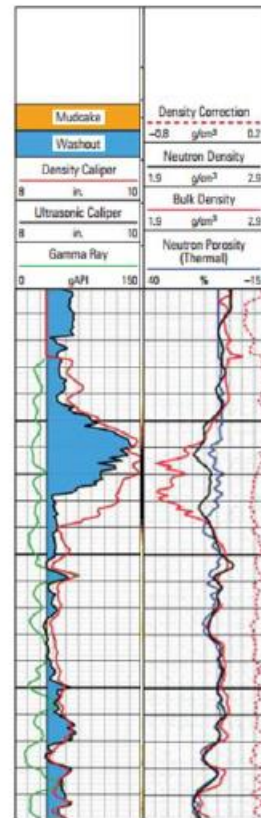


Well Log Reconstruction

Identifying and removing bad data, filling gaps and estimating missing logs using rock physics

Missing Sonic or Density Logs? Poor Log Quality?

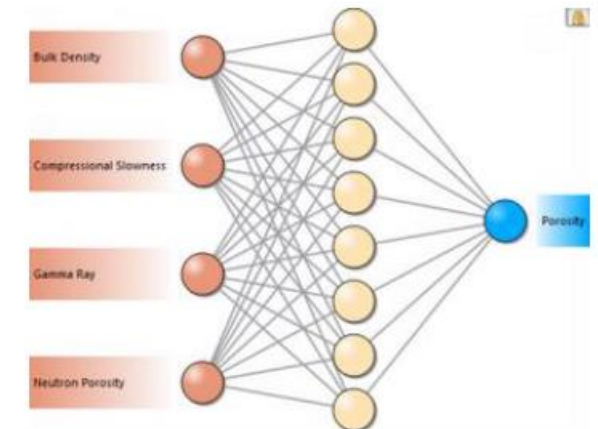
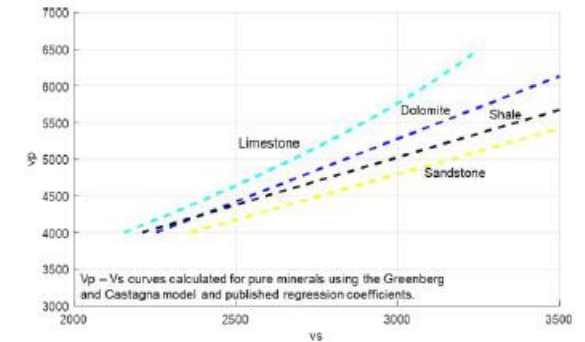
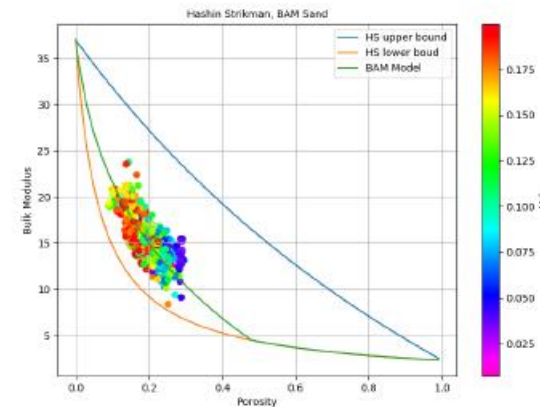
- Spikes
- Washouts
- Cycle Skips
- Mud Invasion
- Outside physical bounds



Replace
missing log
data

Well Logs can be Reconstructed/Modelled using:

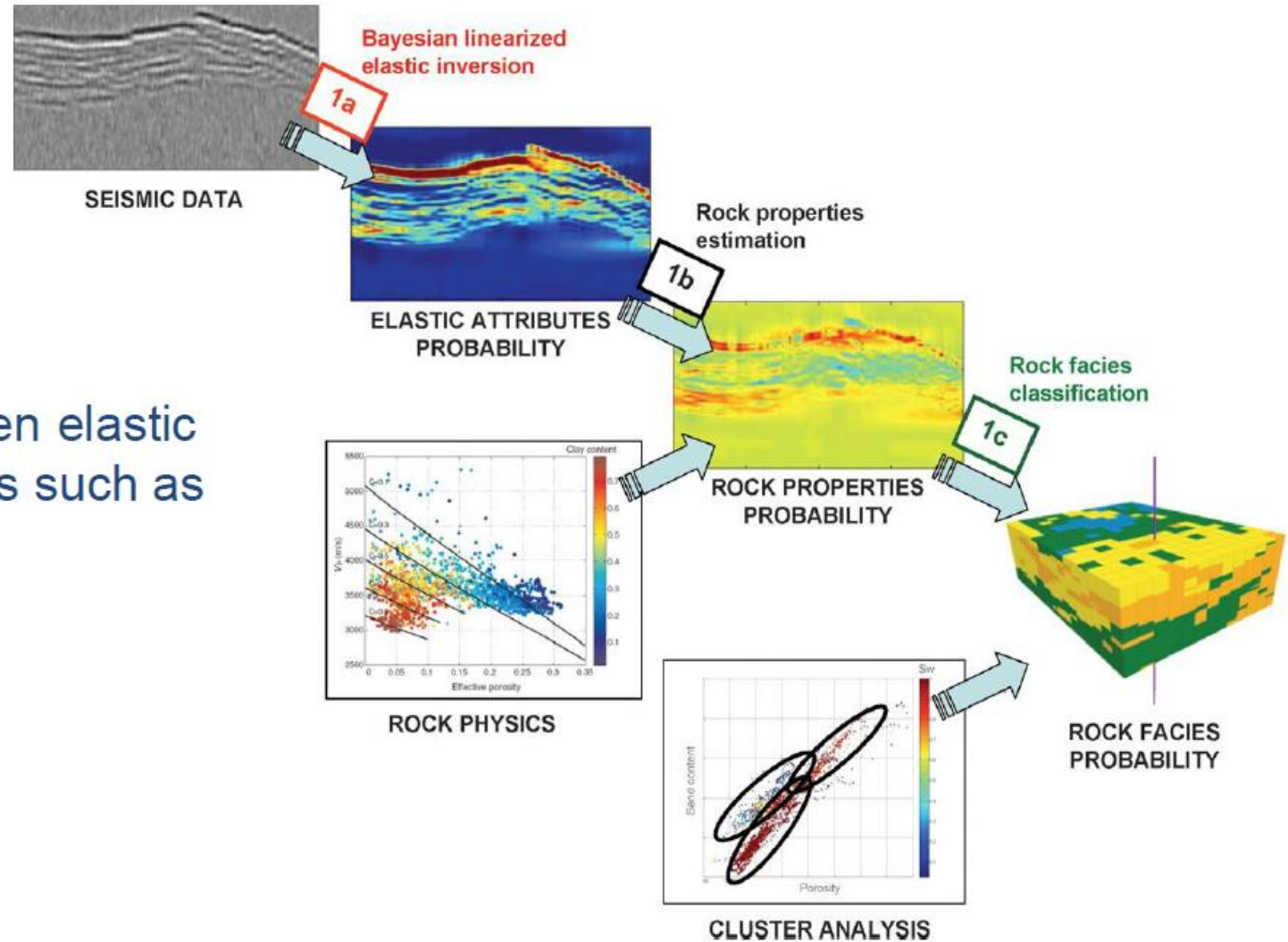
- Empirical Rock Physics models
- Theoretical Rock Physics models
- Multi-Linear Regressions
- Neural Network Techniques



Ref: **The Rock Physics of AVO**
Stanford Rock Physics Laboratory - Gary Mavko

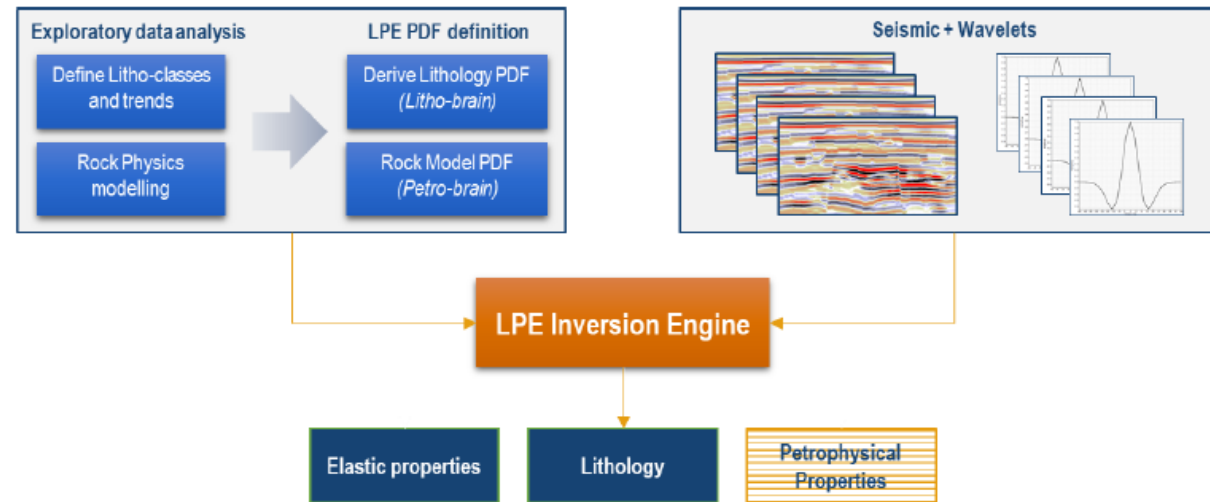
Lithology Prediction

It provides the relationship between elastic properties and reservoir properties such as lithology and porosity.



Litho-Petro-Elastic Inversion (LPE)

Rock Physics modelling and compaction trend analysis of the basin enable the **joint inversion** of seismic amplitudes to lithology units and elastic properties



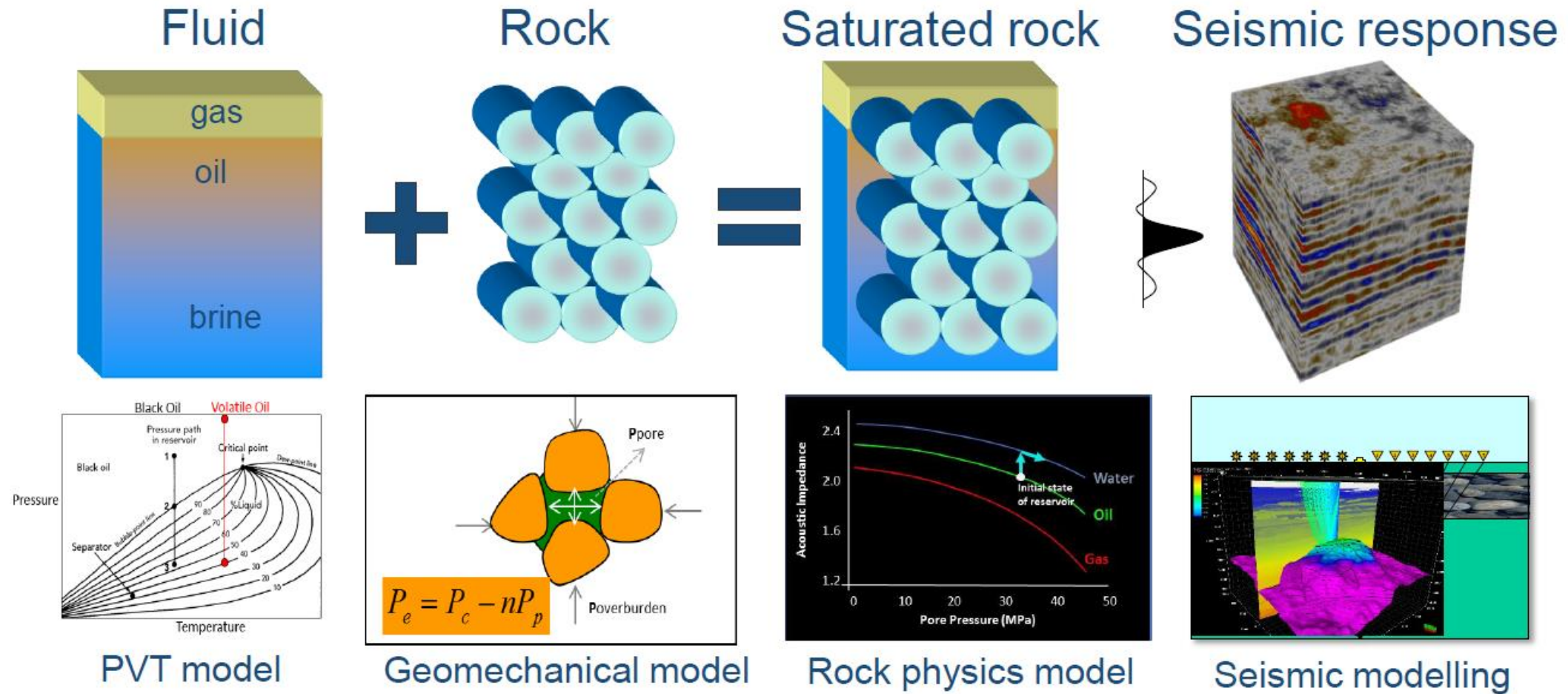
Litho-elastic trends are governed by geological processes

- Mechanical compaction, geochemical alteration, cementation, etc.

Overcomes the challenge of building accurate well-based low frequency prior models in

- Frontier exploration areas, channelized systems, structurally complex areas

Forward Modelling - Simulation to Seismic (Sim2Seis)



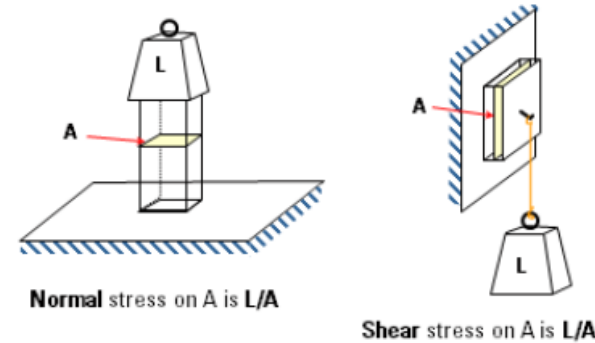
Rock physics concepts

- Definitions of some important elastic moduli
- Different modes or propagation: P waves and S waves
- Velocity relationships
- Empirical rock property relationships

Elasticity concepts – Stress & Strain

STRESS is the *force per unit area* applied to a material or rock

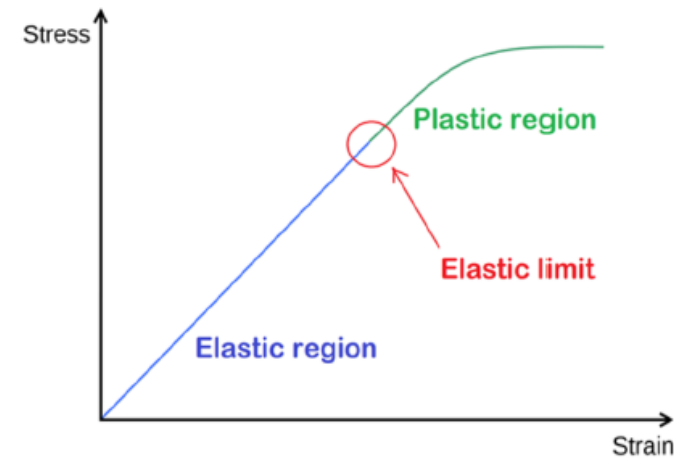
STRAIN is the *deformation* resulting from stress



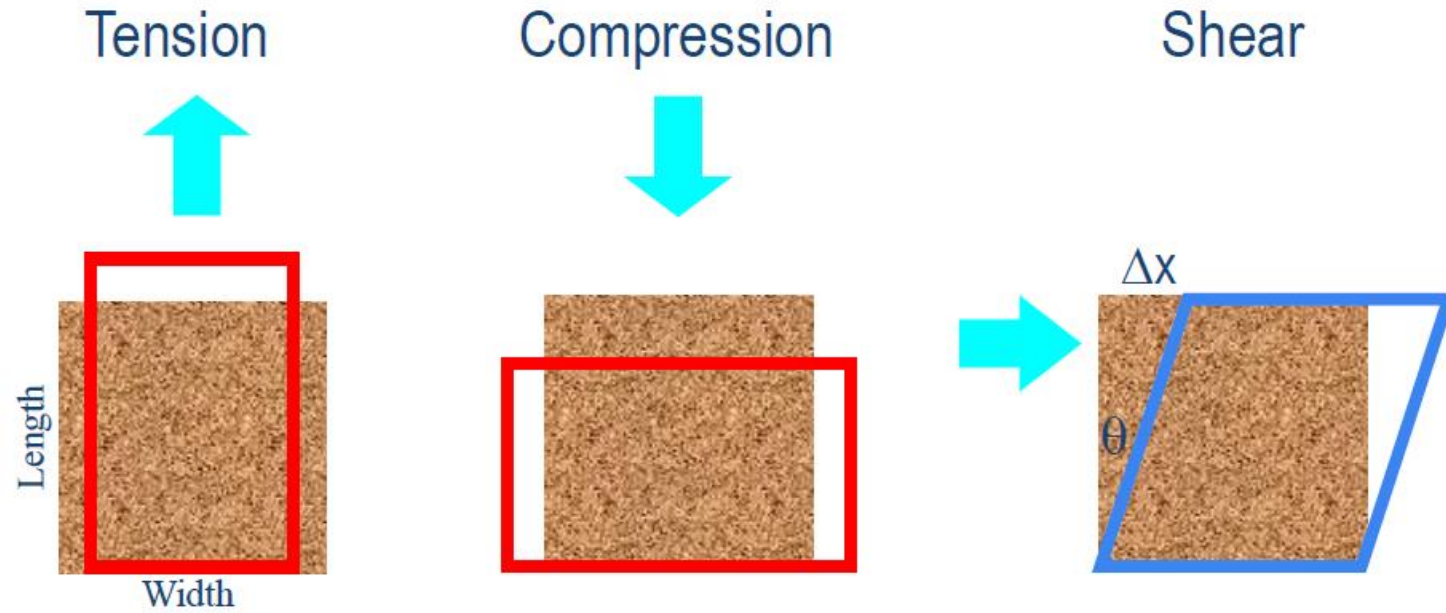
Hooke's law: Strain is directly proportional to the applied stress up to yield point

ELASTIC DEFORMATION is *nonpermanent* deformation: the body returns to its original shape when the stress is released

Elastic Moduli: The linear relationship between stress and strain in the elastic field is defined for any material by its elastic moduli



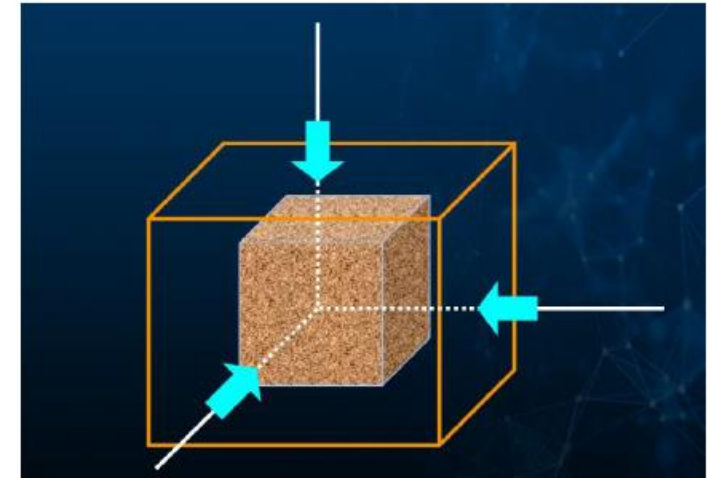
Elasticity concepts – Stress & Strain



$$\text{Longitudinal Strain} = \frac{\Delta \text{Length}}{\text{Length}}$$

$$\text{Transverse Strain} = \frac{\Delta \text{Width}}{\text{Width}}$$

$$\text{Shear Strain} = \frac{\Delta x}{\text{Length}} = \tan \theta$$



$$\text{Volumetric Strain} = \frac{\Delta \text{Volume}}{\text{Volume}}$$

Elasticity concepts – Stress & Strain

Longitudinal Stress = $E \times$ Longitudinal Strain

E = Youngs Modulus

$$P_L = E \Delta L / L$$

Shear Stress = $\mu \times$ Shear Strain

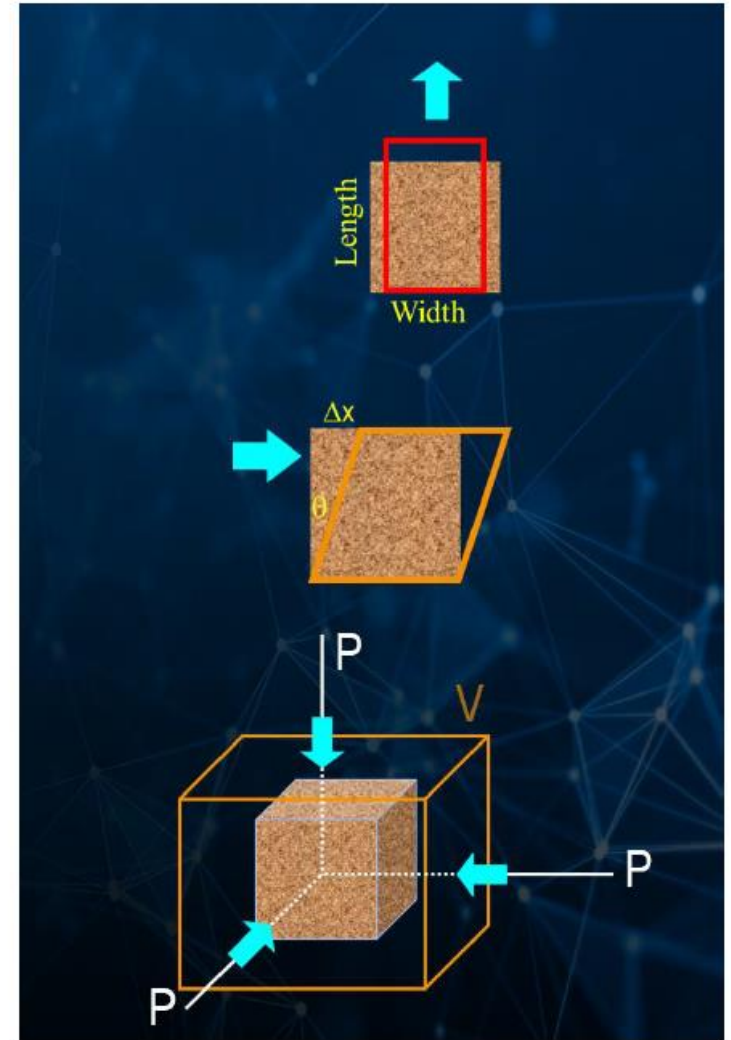
μ = Shear Modulus or Rigidity

$$P_s = \mu \Delta x / y = \mu \tan \theta$$

Volumetric Stress = $K \times$ Volumetric Strain

K = Bulk Modulus or Incompressibility

$$P_V = K \Delta V / V$$

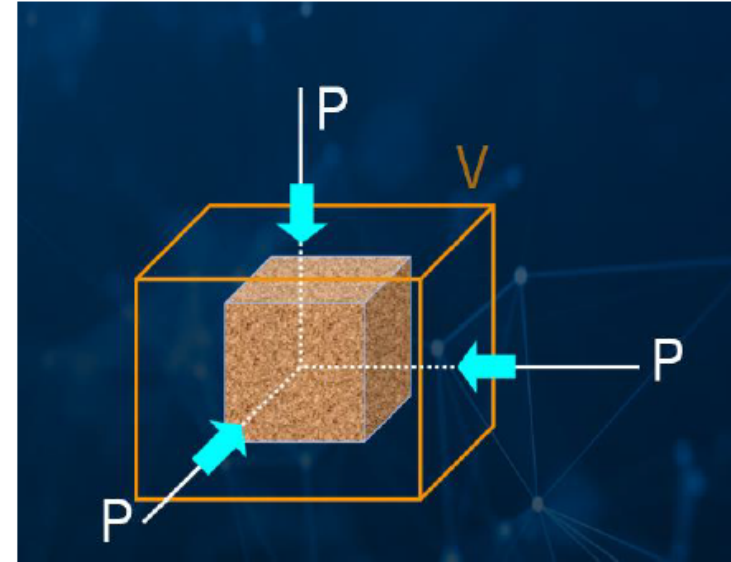


Bulk modulus

Bulk modulus, K , of an isotropic rock is the ratio of hydrostatic stress to the volumetric strain

Can be determined in the lab using dynamic measurements of P- and S- wave velocities and density, or from log data.

For both cases $K = \rho \left(V_p^2 - \frac{4}{3} V_s^2 \right)$



$$P = -K \frac{\Delta V}{V}$$



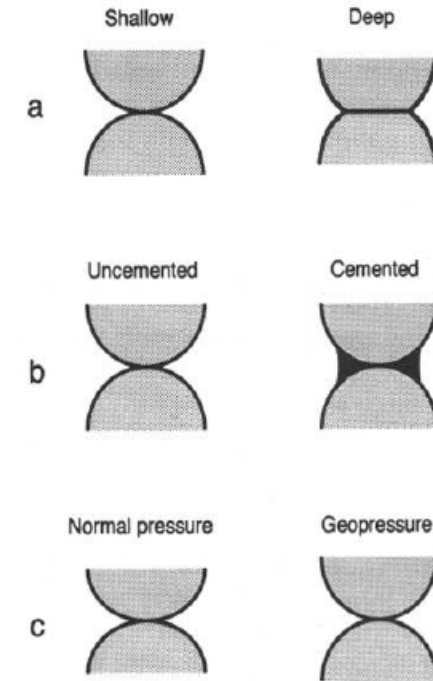
Gpa (typical unit)

Shear modulus

Shear modulus of an isotropic rock is the ratio of the shear stress to shear strain. It can be determined in the lab using dynamic measurements of S-wave velocities and density or from log data

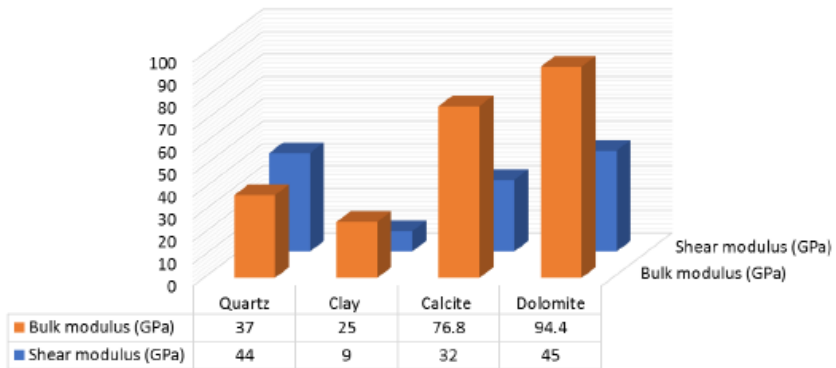
$$\mu = \rho V_s^2$$

The shear rigidity, μ , relates to the resistance to a shearing - or change of shape. Shear rigidity depends on the grain contacts, especially cementation, shape, sorting and pressure

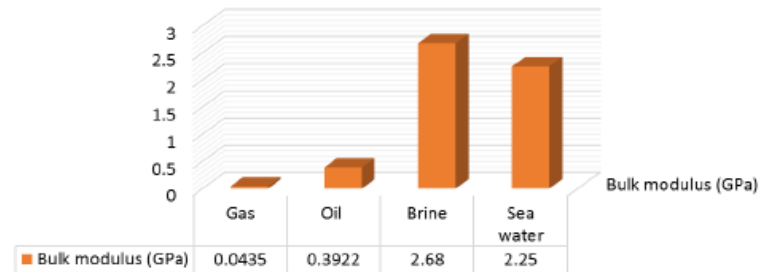


Bulk and Shear moduli

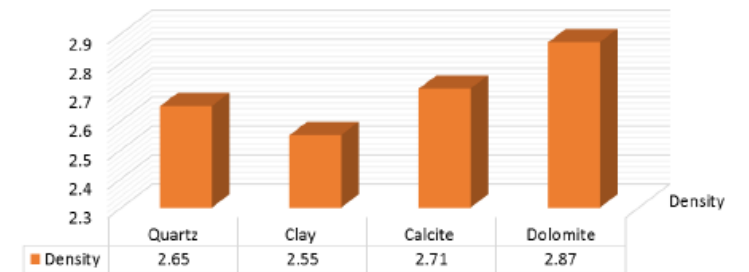
Bulk and shear moduli (GPa)



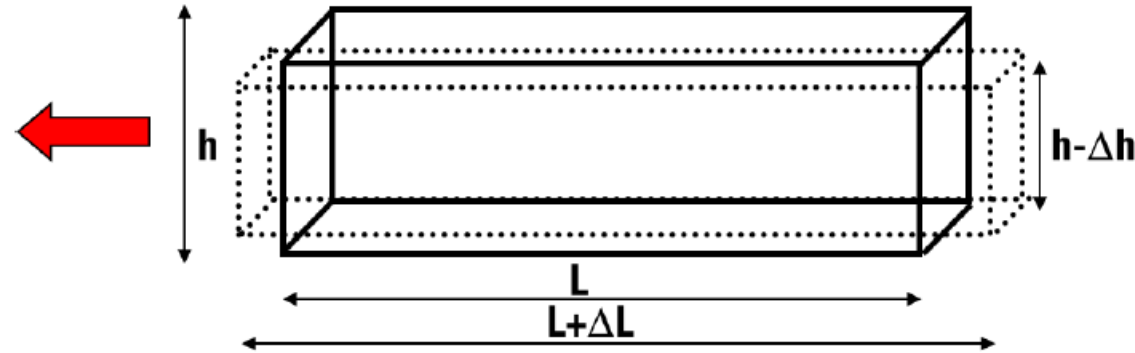
Bulk modulus (GPa)



Density

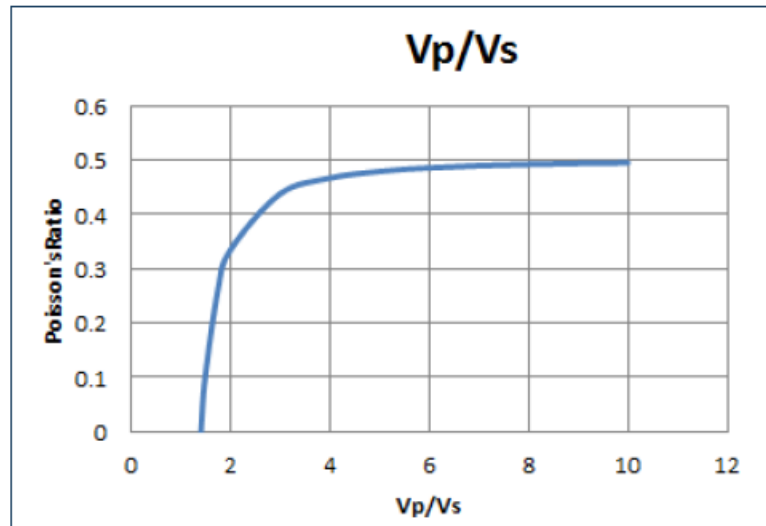


Elastic constants: Poisson's ratio



$$\sigma = -\left(\frac{\Delta h}{h}\right) / \left(\frac{\Delta L}{L}\right)$$

$\Delta L/L$ is the longitudinal strain
 $\Delta h/h$ is the transverse strain
 σ is Poisson's ratio



$$\sigma = \frac{\left(\frac{V_p}{V_s}\right)^2 - 2}{2 \left[\left(\frac{V_p}{V_s}\right)^2 - 1 \right]}$$

P wave and S wave velocity relationships

P wave velocity

$$V_p = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}$$

S wave velocity

$$V_s = \sqrt{\frac{\mu}{\rho}}$$

P/S velocity ratio

$$\frac{V_p}{V_s} = \sqrt{\frac{K}{\mu} + \frac{4}{3}}$$

P and S impedances

$$Z_p = V_p \cdot \rho \quad Z_s = V_s \cdot \rho$$

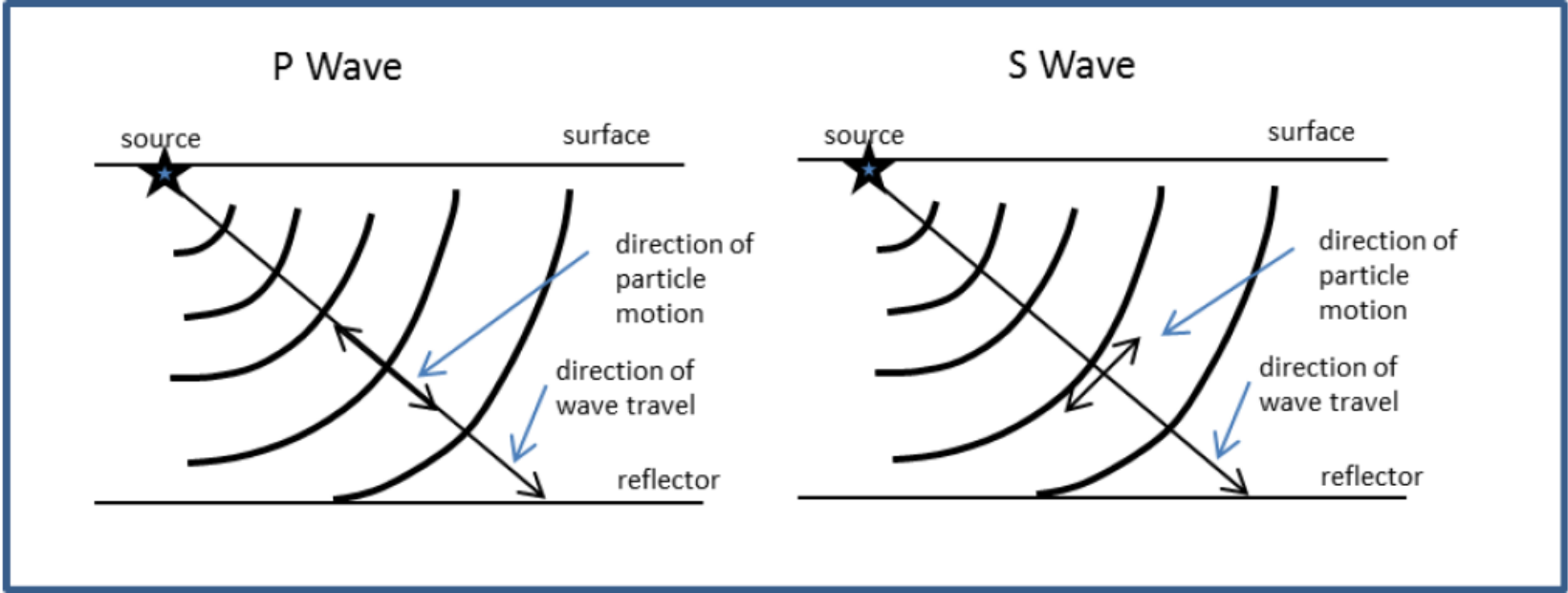
K (bulk modulus): rock deformation to a compressional stress

μ (shear modulus): rock deformation to a shear stress, not influenced by pore fluid

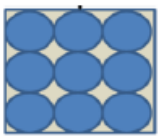
ρ is the density

Elastic moduli

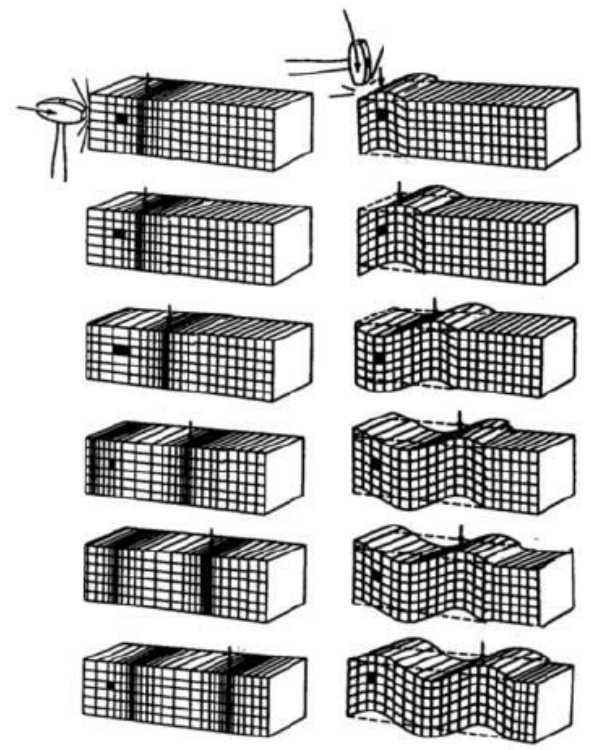
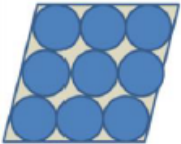
Modes of wave propagation: P waves and S waves



P wave mode: Particle motion along the direction of travel (Compression)

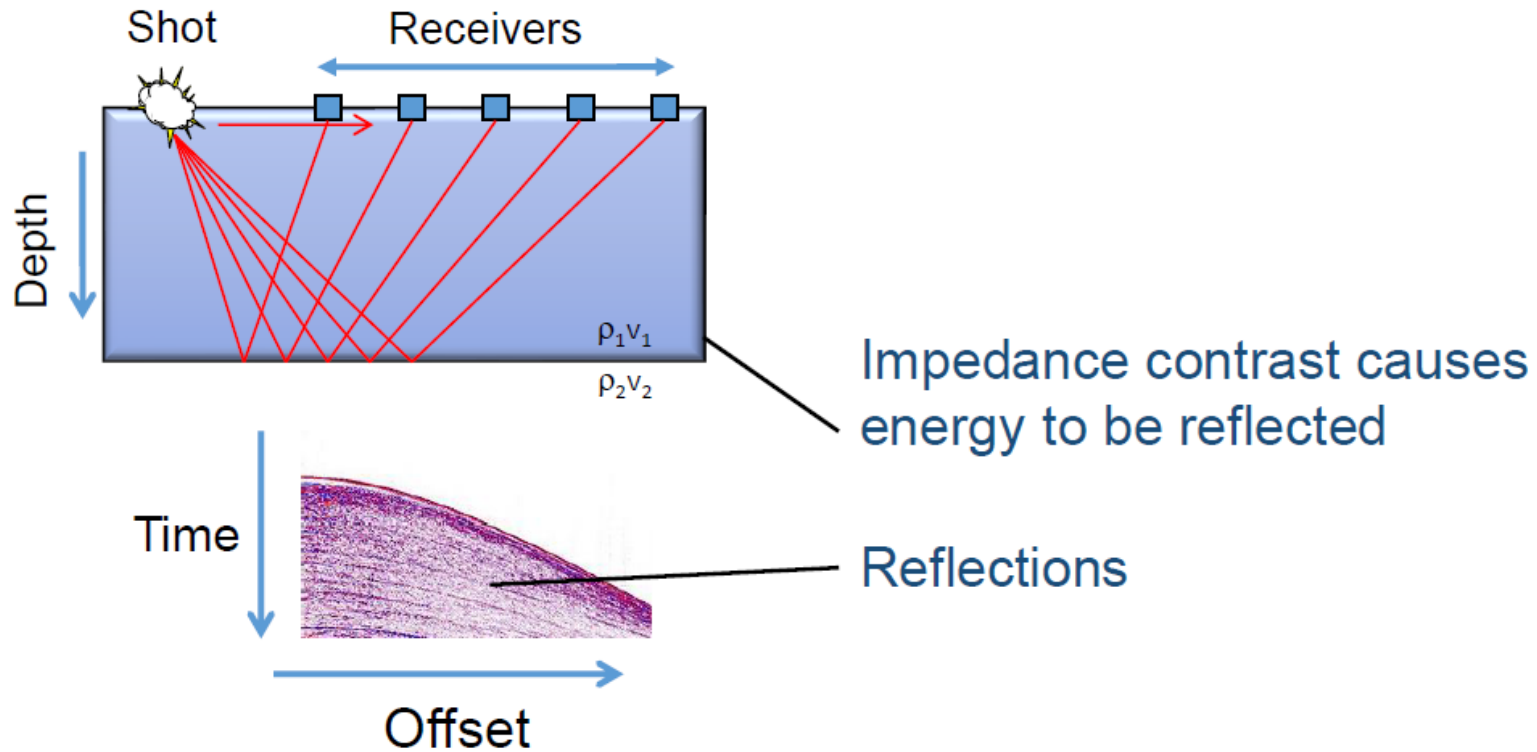


Shear mode: Motion perpendicular to the direction of travel (Shear motion) > Slow



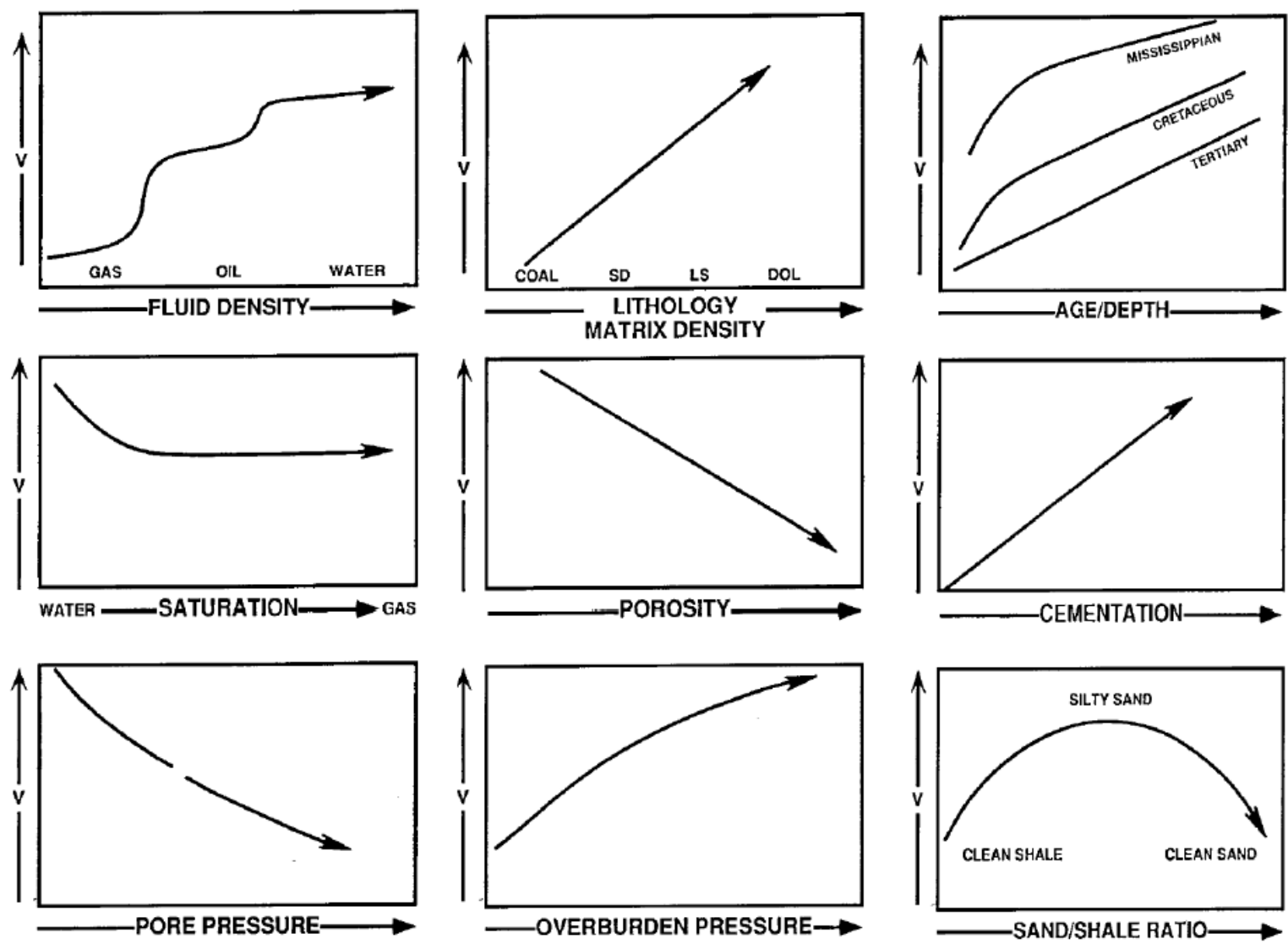
(Sheriff and Geldart, 1995)

Basic seismic experiment



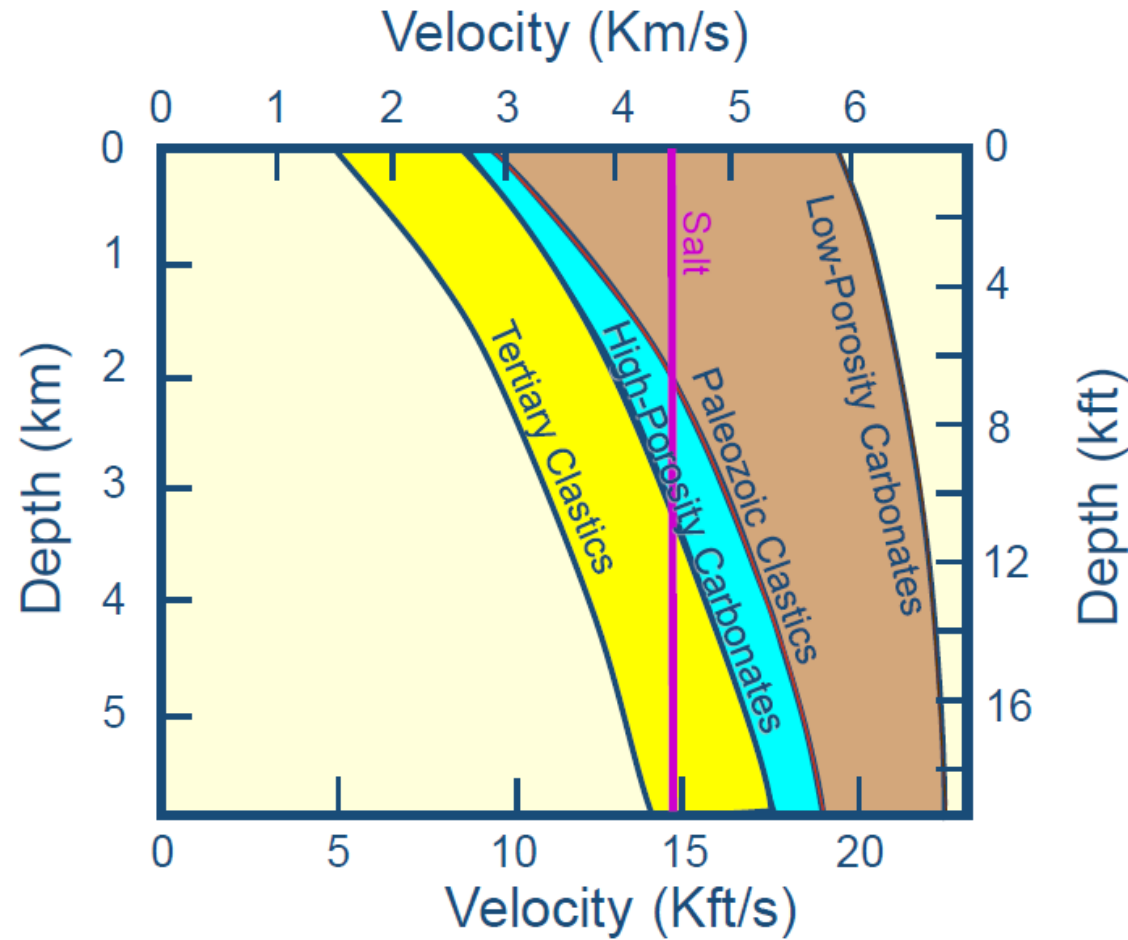
Acoustic impedance $AI = \text{density } \rho * \text{velocity } V$
> Change in velocity or density causes reflections

Factors affecting velocity V_p



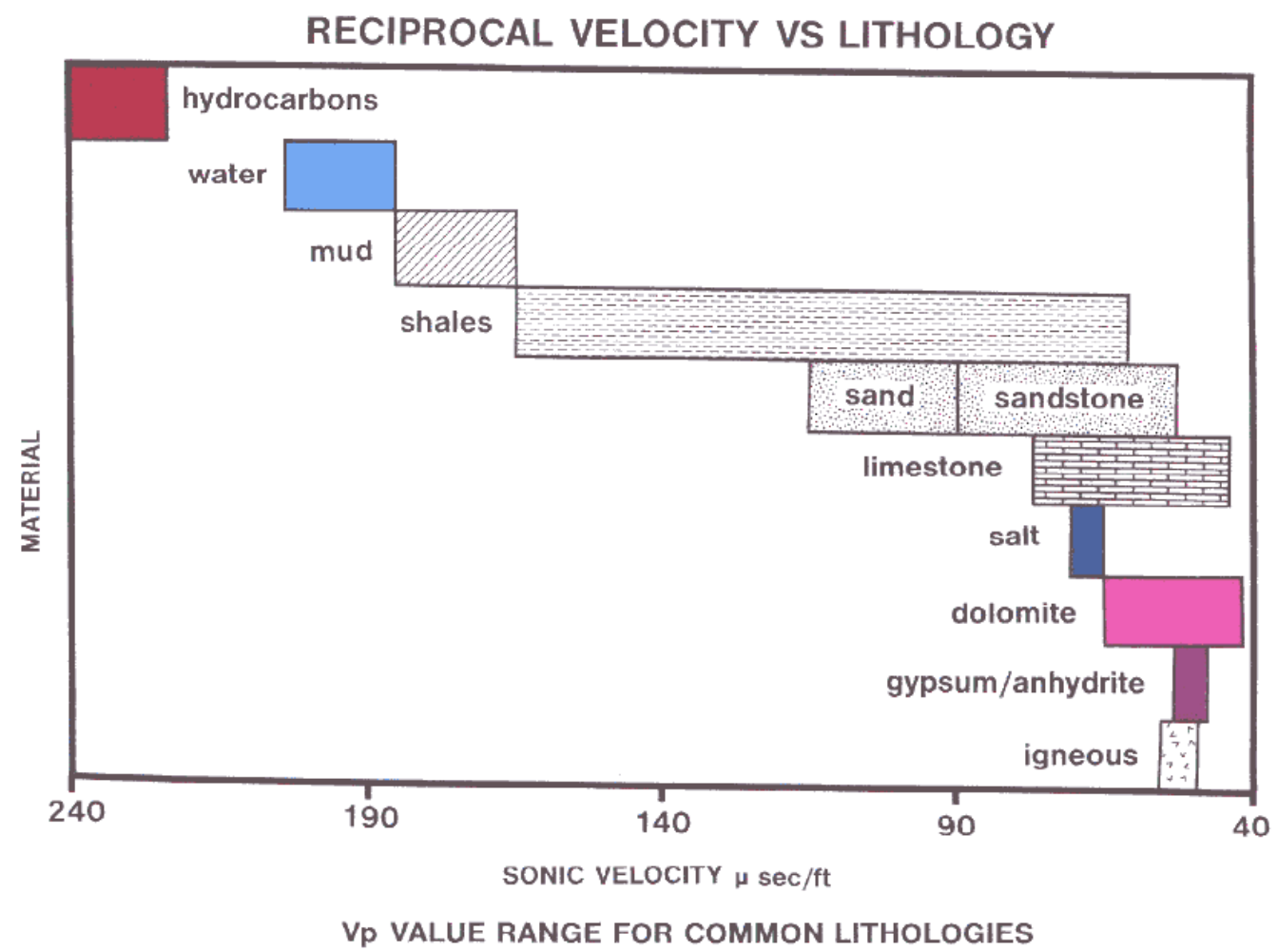
The effects of various factors
on seismic wave velocity
(Hiltermann, 1977)

Velocity range for rocks

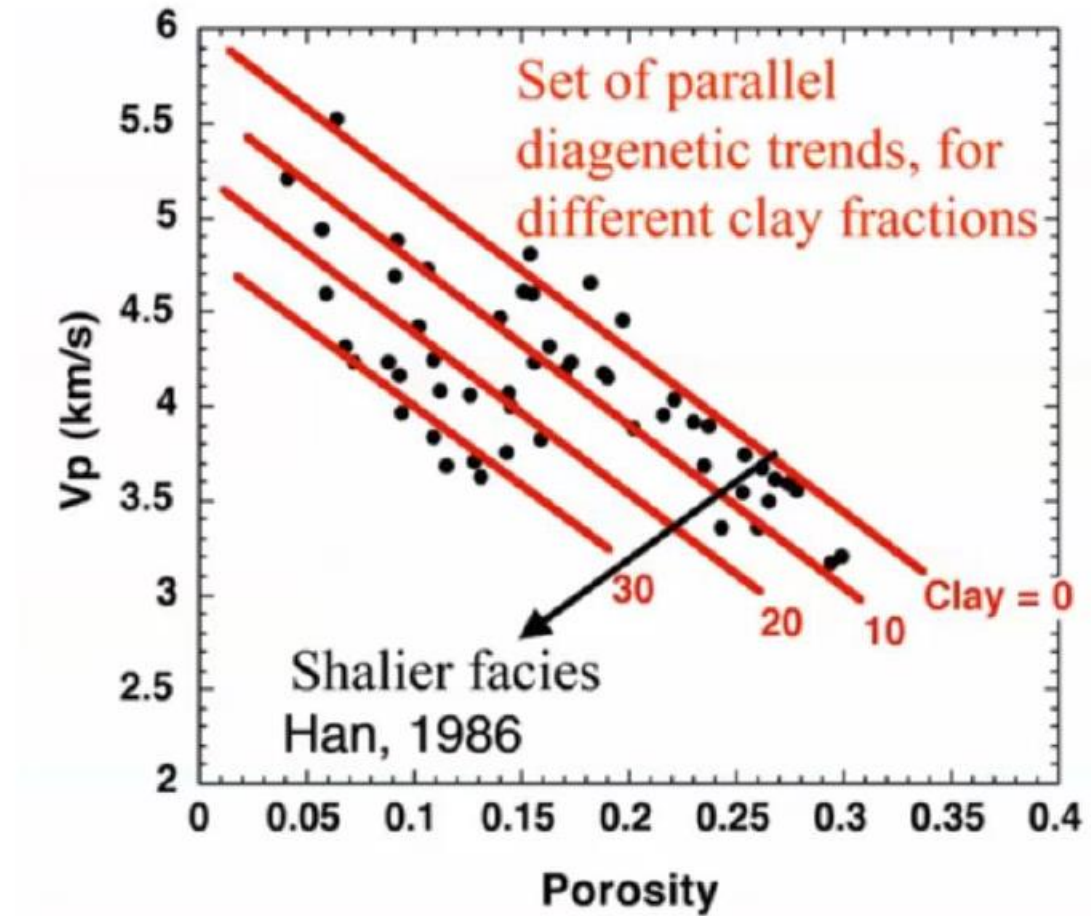


Variation of velocity with depth for various types of lithology (Adapted from Sheriff, 1976)

Factors affecting velocity

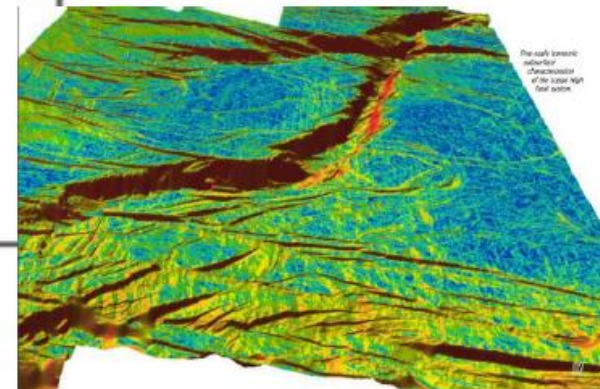
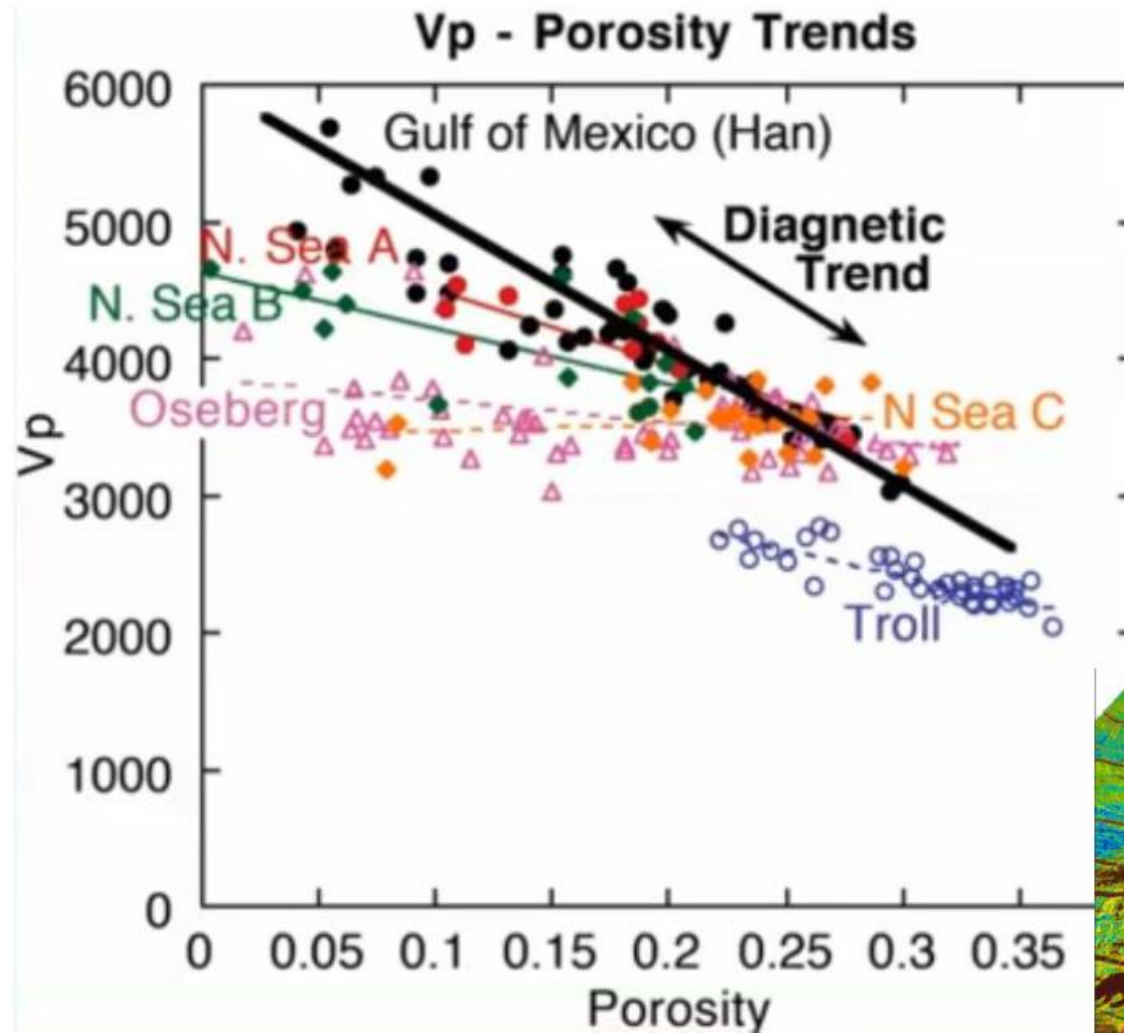


Scatter around the diagenetic trend due to clay



D-H Han, 1986, PHD thesis, Stanford University

Complex nonunique trends



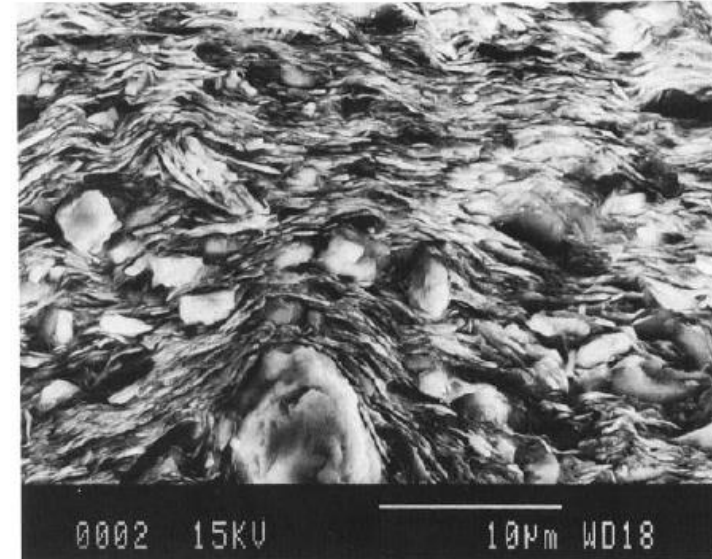
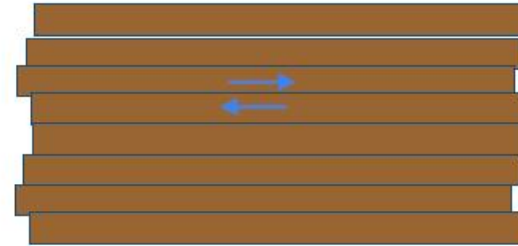
Mavko, 2006

Clays/shales and V_p/V_s

Sub-horizontal platelets and internal structure

Weak shear strength between layering,
compressive strength remains the same

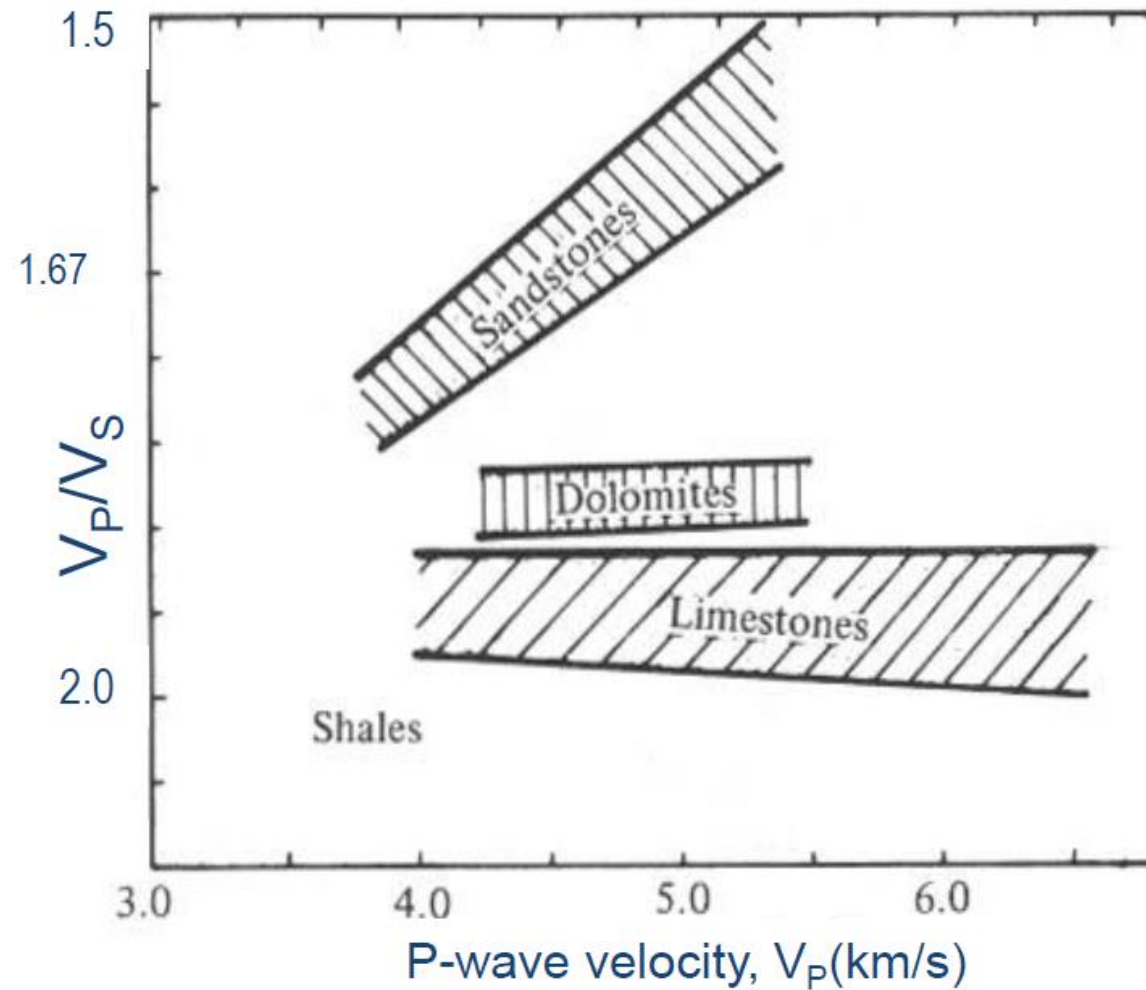
Low μ and therefore a high V_p/V_s



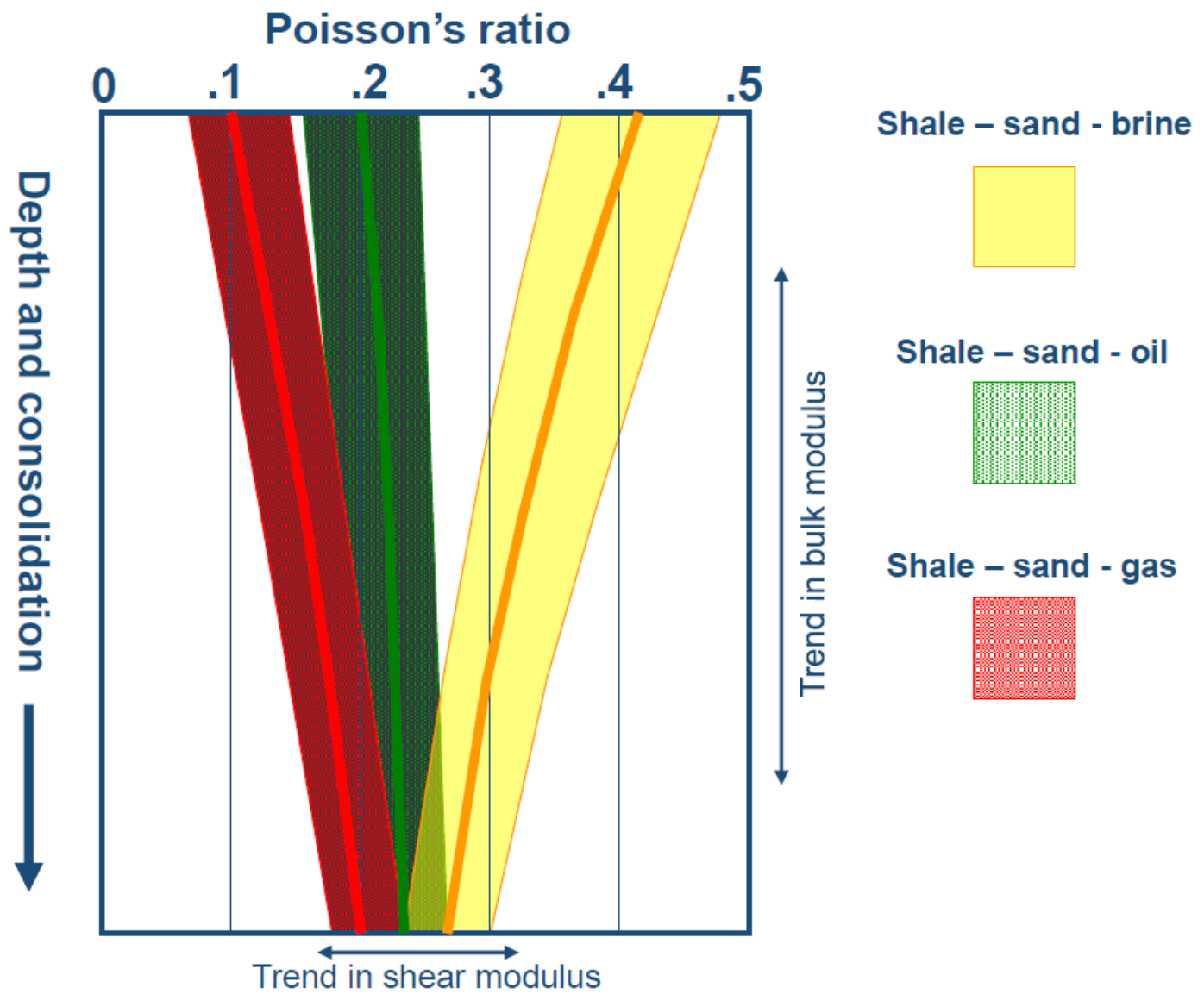
Scanning electron microphotograph of a shale.
The platey particles are clay minerals while the
larger, nearly spherical, particles are silt.

Picture from my book/Hornby pdf

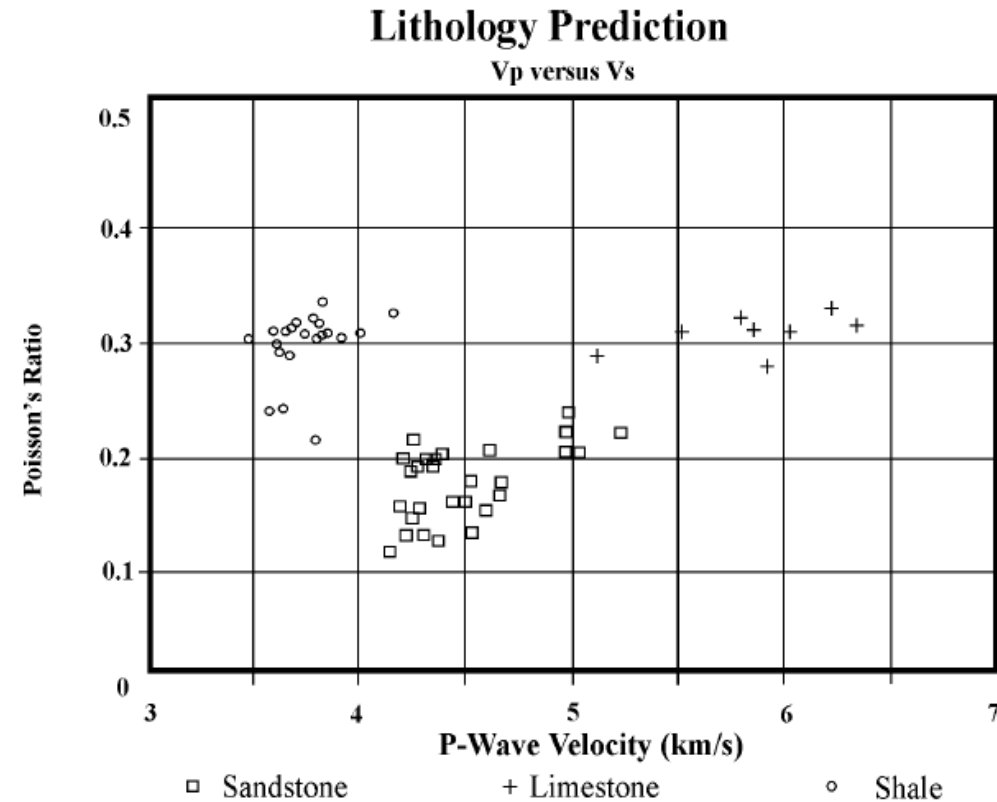
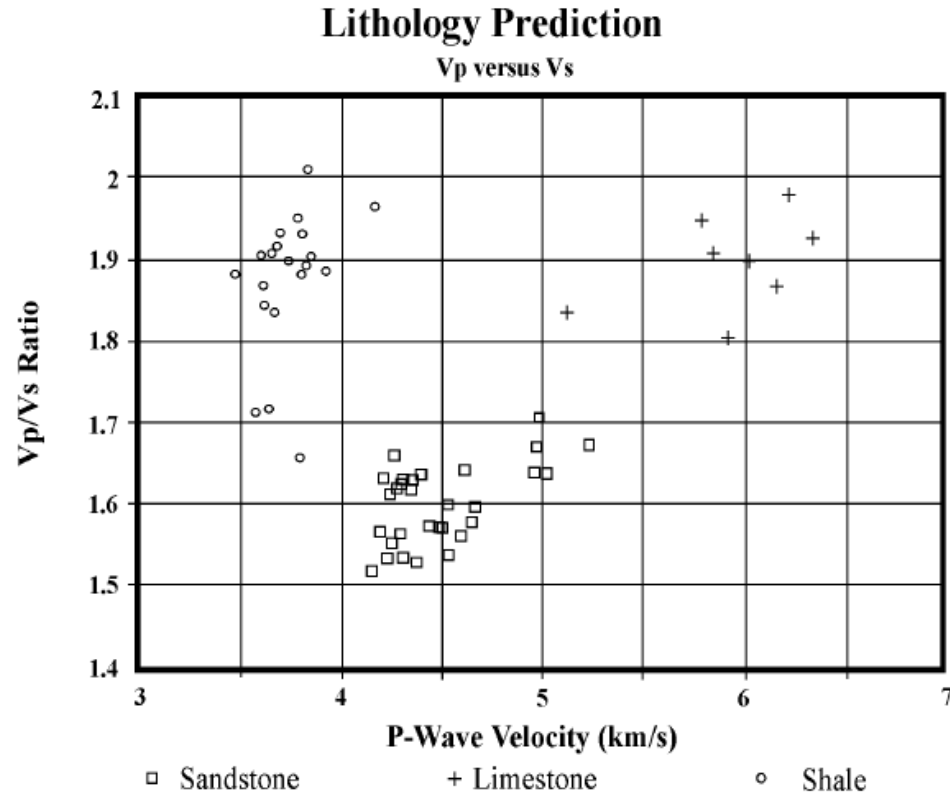
V_p/V_s as a lithology indicator



Poisson's Ratio and depth



Poisson's Ratio and VpVs

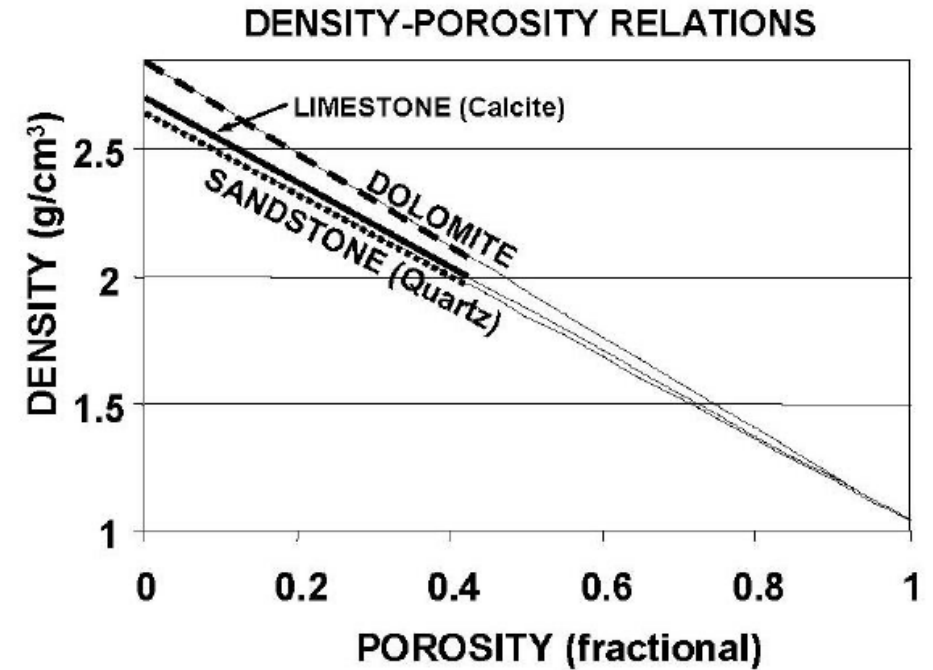


Lithology prediction based on the Vp-Vs and Poisson's ratio crossplot
with P-wave velocity (Russel, 1998)

Density

The density related to various factors, including:

- 1) The type and number of minerals and their percentages
- 2) **The porosity of the rock**
- 3) The types of fluids filling the pore space
- 4) Depth

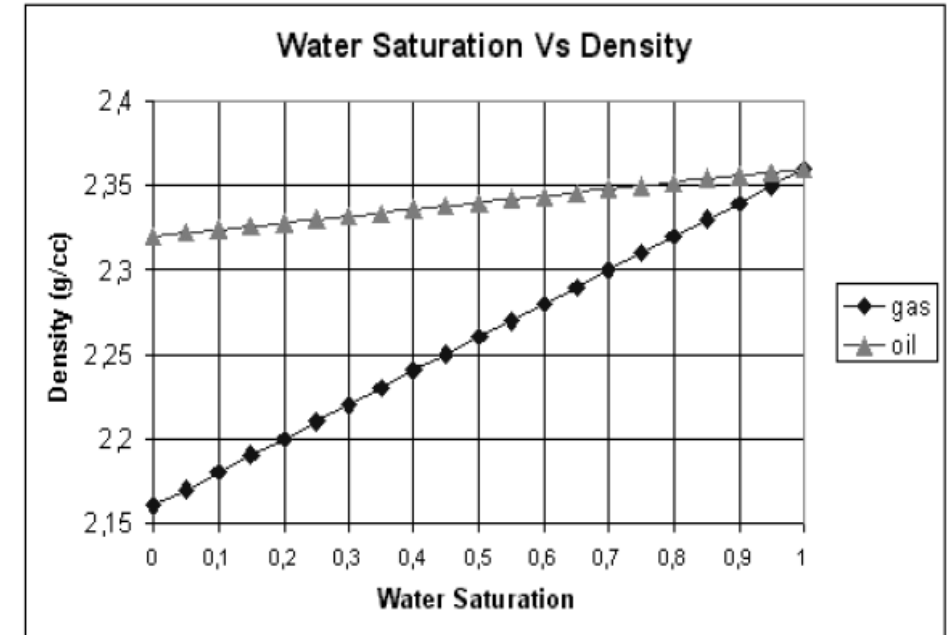


In general, the density increases and porosity decreases monotonically with depth

Density

The density related to various factors, including:

- 1) The type and number of minerals and their percentages
- 2) The porosity of the rock
- 3) **The types of fluids filling the pore space**
- 4) Depth



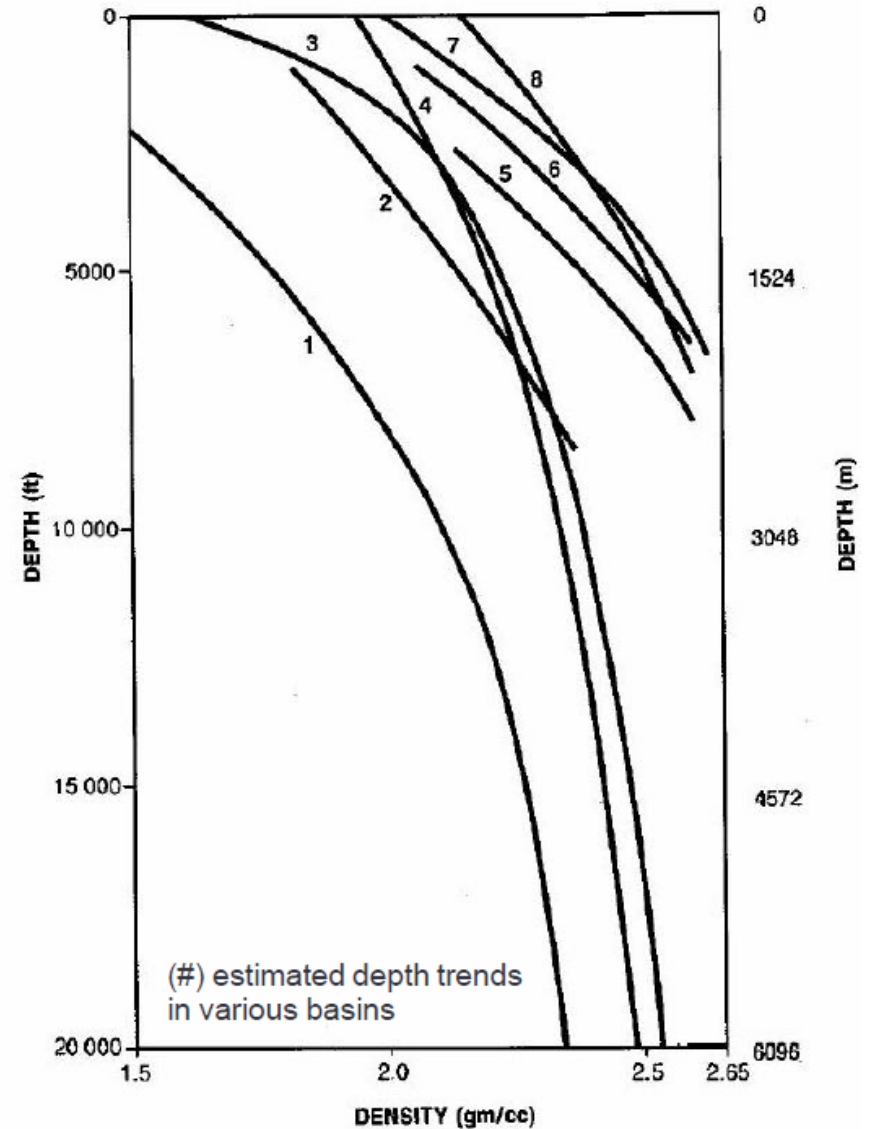
Wyllie's equation applied to a gas and oil reservoir. This model uses parameter : $\Phi = 20\%$, matrix density = 2.7 g/cc, gas density = 0.001 g/cc, oil density = 0.8 g/cc

Density

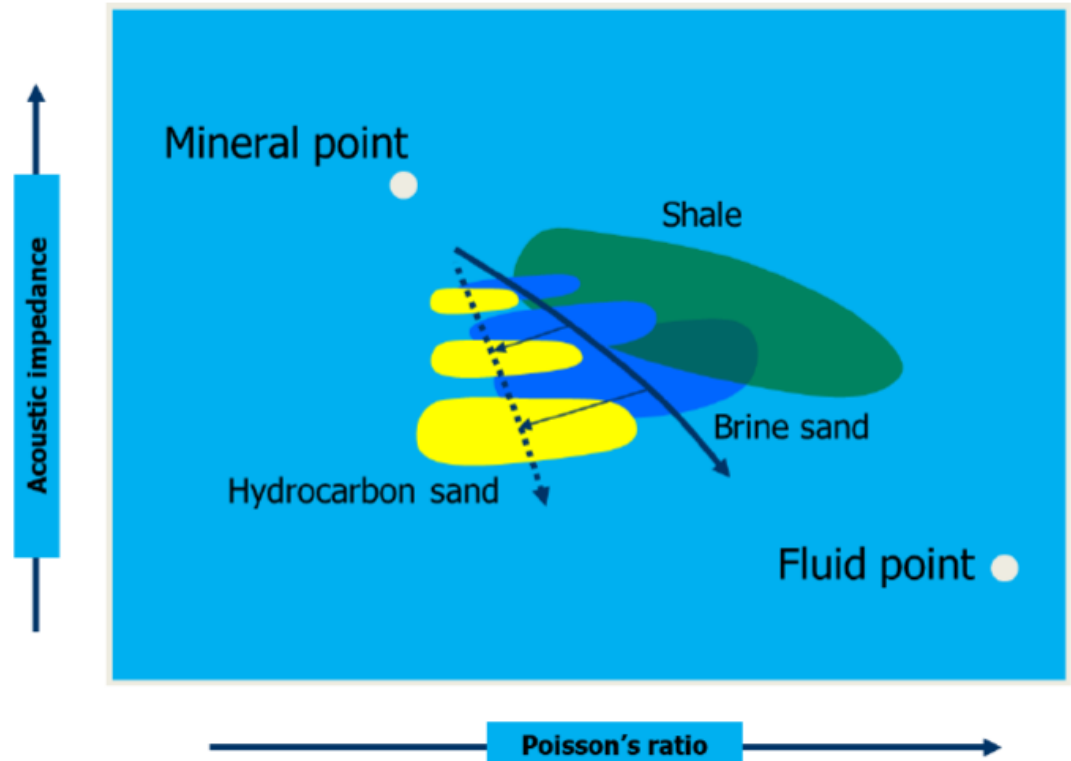
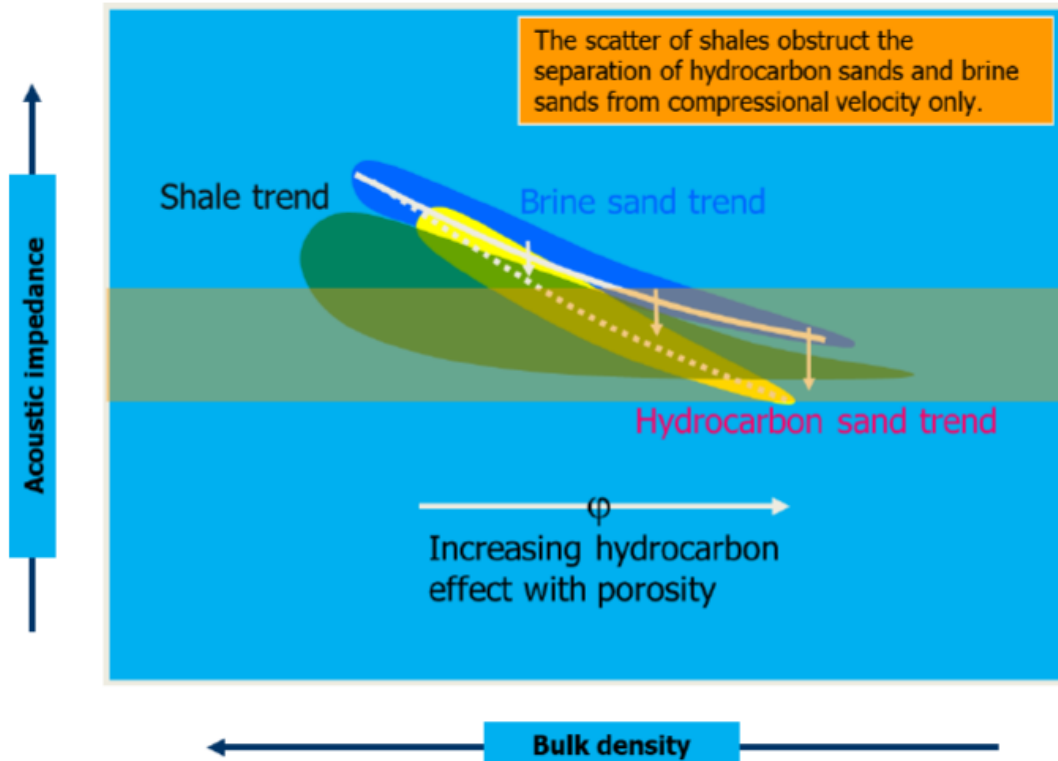
The density related to various factors, including:

- 1) The type and number of minerals and their percentages
- 2) The porosity of the rock
- 3) The types of fluids filling the pore space
- 4) **Depth**

This is expected since differential **pressure** usually **increases** with depth. As pressure increases, grains will shift and rotate to reach a **more dense packing**. Diagenetic processes such as **cementation** work to fill the pore space. These kinds of curves are often fit with exponential curves in depth to define the local compaction trend.



Cross plot analysis, conceptual cross plot



Rock Physics | Elastic Properties and crossplots

- Calculate V_p , V_s , A_1 , V_p/V_s , from well logs
- Create the following cross plots:
 1. V_p versus V_s
 2. V_p versus Density
 3. A_1 versus V_p/V_s
 4. V_p versus Depth
- Discussion - do you see any trends in the plots?
- Color the cross plots of A_1 versus V_p/V_s by:
 1. Porosity
 2. Water saturation
 3. Volume of shale

Rock physics relations

Equations used in rock-physics are either theoretical or empirical:

- **Theoretical relationships** start with underlying physical principals and attempt to propose a universal relationship. May break down when (hidden?) assumptions are violated.
- **Empirical relationships** are derived from experiment. Work well on an individual basis but are often difficult to compare from one project to another.

Empirical relationships

- Several empirical relationships have been derived over the years that relate density to P wave velocity and P wave to S wave velocities.
- These relationships are useful when actual measurements are unavailable (specific logs not run, poor log quality, old wells).
- These relationships also can be useful for modeling studies in areas far away from well control or in basins where there might be no well control and velocity information is available only from seismic data.

But they should be **calibrated** if there is relevant data!

Empirical relationships: Velocity-Density

- Gardner relationship (1974) relates P wave velocity to density through the formula:

$$\rho = 0.31V^{0.25}$$

Density is in grams/cc.

Velocity is in meter per second

The constant 0.31 depends on the units being used

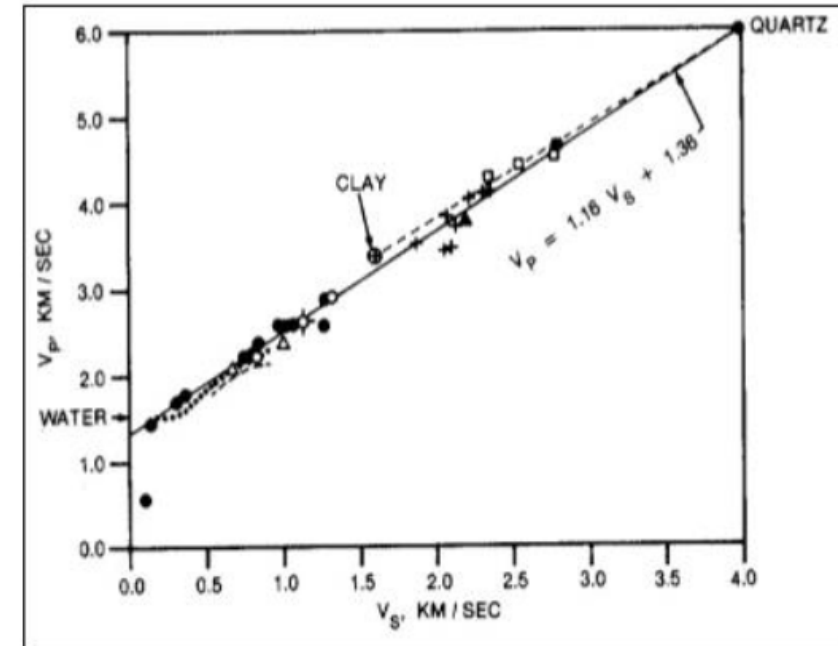
The relationship was derived for clastic sediments that were brine filled

Empirical relationships: Castagna mudrock for V_p - V_s

- An early relationship published by Castagna (Geophysics, 1985) from work at ARCO relates V_p to V_s . It is referred to as the Castagna or Arco mudrock line.

$$V_p = 1.16V_s + 1360 \text{ m/s}$$

This relationship was derived from water saturated sands, shales, and siltstones.



*ARCO's original mudrock derivation
(Castagna et al, Geophysics, 1985)*

Greenberg and Castagna Vp - Vs

- Castagna's work was extended with additional data (1992).
- Relationships were derived for brine saturated rocks: sands, shales, limestones, and dolomites.
- These results are known as the Greenberg and Castagna relationships.

Sandstone (km/sec): $V_s = -0.856 + 0.804V_p$

Limestone (km/sec): $V_s = -1.031 + 1.017V_p - 0.055V_p^2$

Dolomite (km/sec): $V_s = -0.078 + 0.583V_p$

Shale (km/sec) : $V_s = -0.867 + 0.770V_p$

Note: Any porosity is assumed brine filled

Vs estimation: Greenberg and Castagna (GC)

- The shear wave velocity is approximated by a simple average of the arithmetic and harmonic means of the constituent pure lithology shear velocities.

$$V_s = \frac{1}{2} \left\{ \left[\sum_{i=1}^L X_i \sum_{j=0}^{N_i} a_{ij} V_p^j \right] + \left[\sum_{i=1}^L X_i \left(\sum_{j=0}^{N_i} a_{ij} V_p^j \right)^{-1} \right]^{-1} \right\}$$

L = number of pure monomineralic lithological constituents

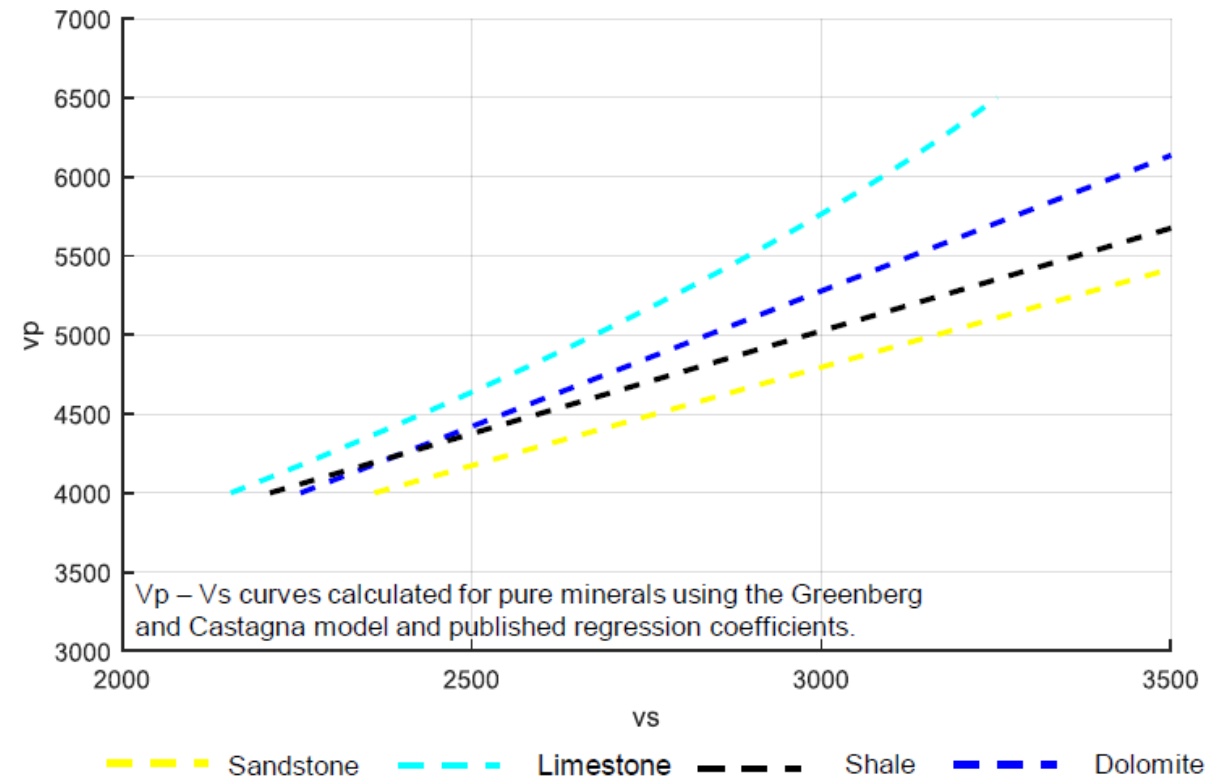
X_i = volume fractions of the minerals

a_{ij} = empirical regression coefficients

N_i = order of the polynomial for constituent i

V_p = P-wave velocity on brine saturated rock

V_s = Estimated S-wave velocity



Effective media theory

Theoretical models based on **effective media theory** that attempts to describe the effective moduli of rocks and sediments by characterizing the behavior of heterogeneous rocks according to the properties of the constituents: the matrix and the pore filling material

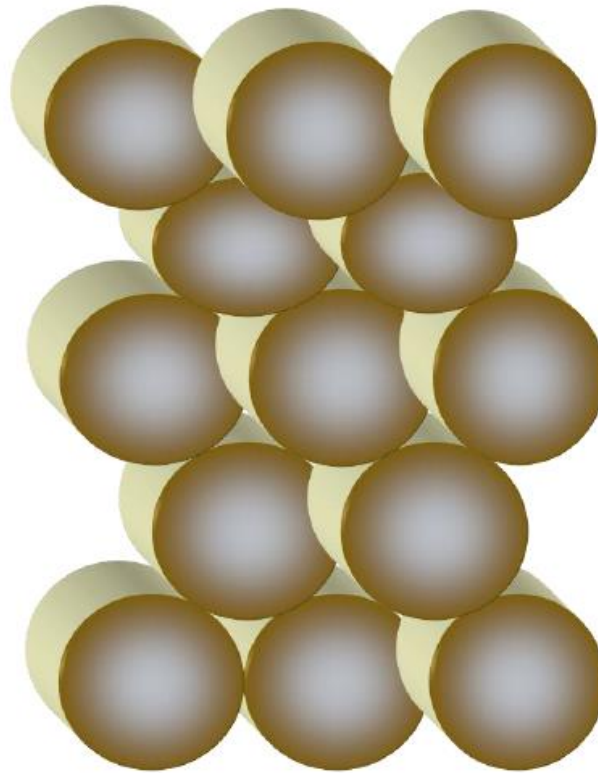
To be able to model or estimate the effective elastic moduli of rocks in terms of the properties of the various constituent minerals and pore fluids, one must incorporate

- the individual elastic moduli of the constituents
- the volume fractions of the constituents
- geometric details of how the various constituents are arranged

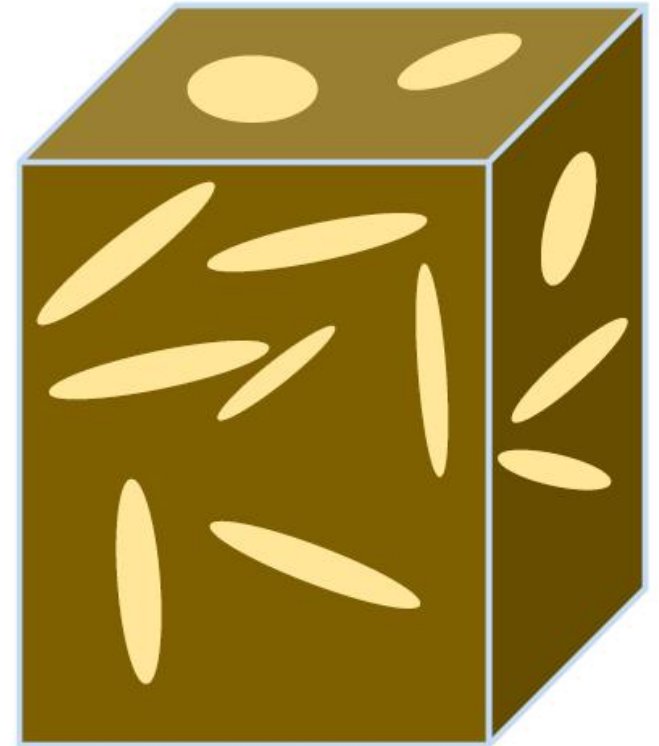


Theoretical models with idealized geometries

The geometric details are the most difficult to know or measure. If we ignore (or don't know) the details of geometry, then the best we can do is to make assumptions



Contact theories



Inclusion theories

Idealized geometries

Theoretical models with idealized geometries

In practice the geometric details of rock and sediment cannot be adequately incorporated into a theoretical model. Attempts always lead to approximations and simplifications, some better than others.

A valuable approximation consists in estimating upper and lower bounds on the moduli or velocities. The bounds are powerful and robust tools:

- rigorous upper and lower limits on the moduli, given the composition

If you find that your measurements fall outside the bounds, then you have made a mistake in velocity, volume fractions, or composition!

Elastic Bounds

One can imagine defining a power law average of the constituents

$$M^\alpha = \sum_{i=0}^n f_i M_i^\alpha$$

f_i fraction of the i th constituent
 M_i the elastic modulus of the i th constituent

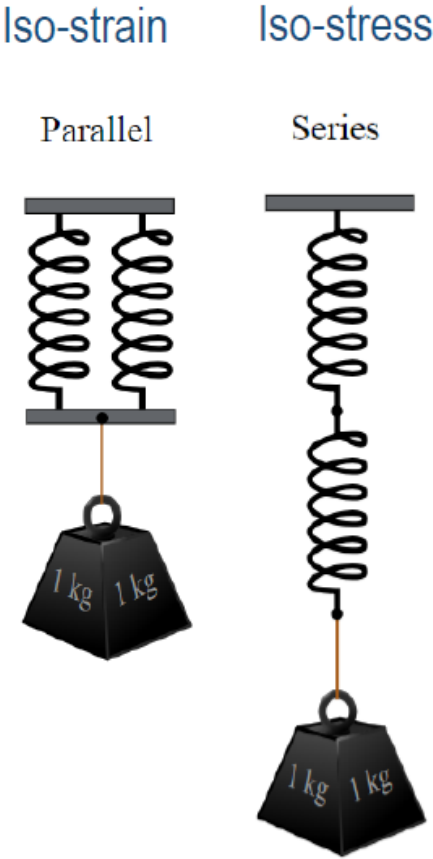
α is the power constant between -1 to +1

Special cases are Voigt average when $\alpha = 1$ and Reuss average when $\alpha = -1$

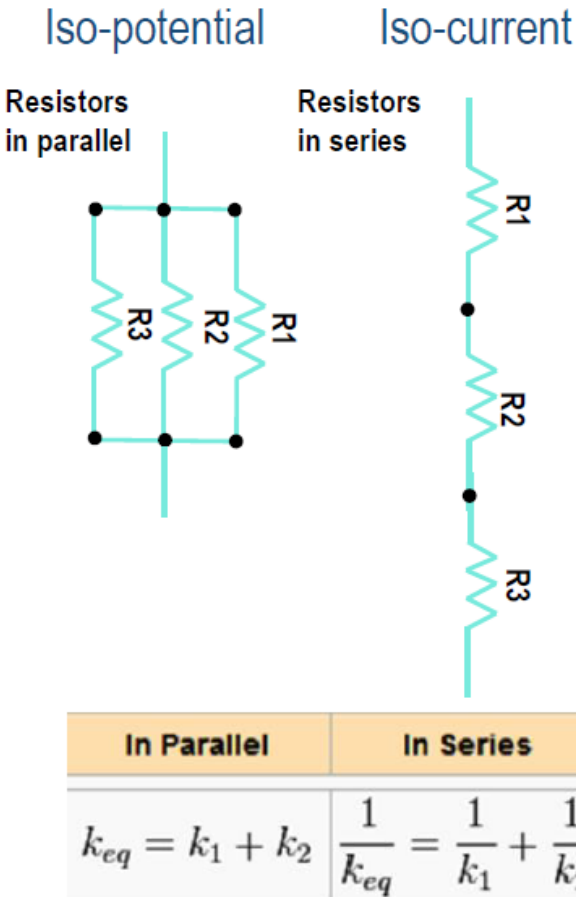
Note: the Voigt-Reuss bounds produces an anisotropic representative of geometry

Geometrical interpretation

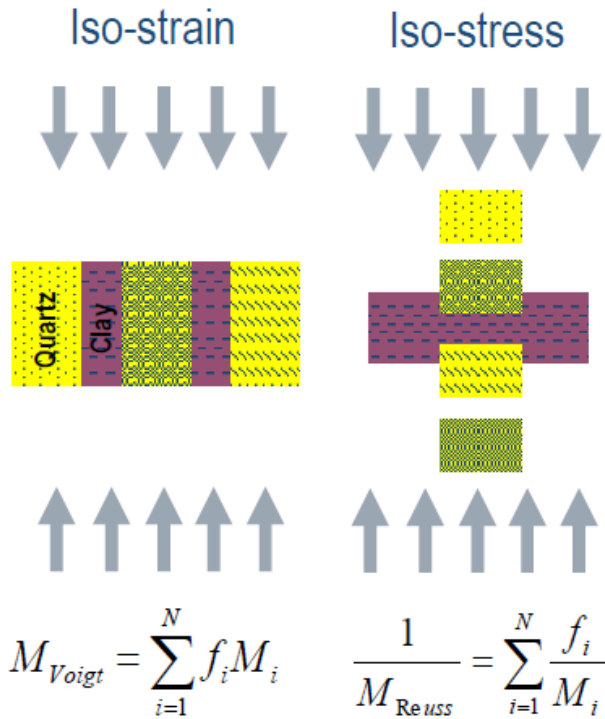
Elasticity



Resistivity



Rock Physics bounds



Has nature has done us a favour?

Voigt upper bound – Arithmetic Average

$$M_v = f_Q M_Q + f_F M_F + f_w M_w + f_o M_o + f_g M_g + \dots$$

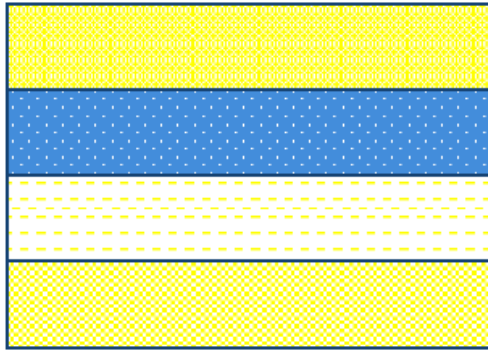
Reuss lower bound – Harmonic Average

$$\frac{1}{M_R} = \frac{1}{f_Q M_Q} + \frac{1}{f_F M_F} + \frac{1}{f_w M_w} + \frac{1}{f_o M_o} + \frac{1}{f_g M_g} + \dots$$



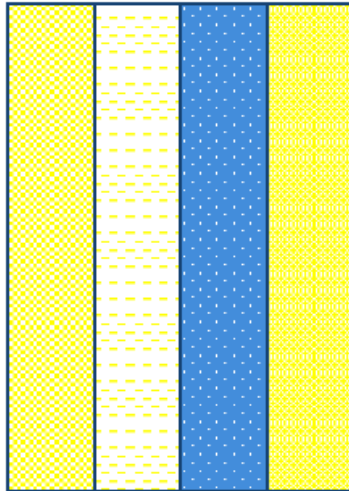
Mixing of solids

Reuss



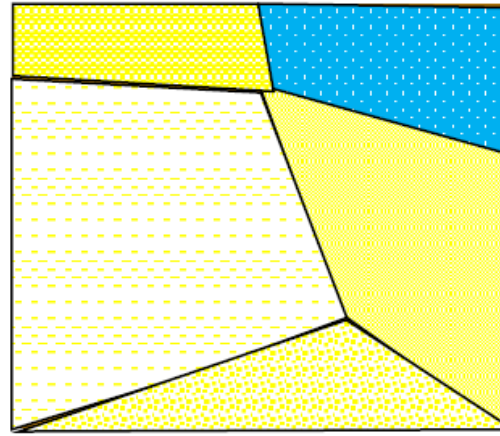
$$\frac{1}{M_{\text{Reuss}}} = \sum_{i=1}^N \frac{f_i}{M_i}$$

Voigt



$$M_{\text{Voigt}} = \sum_{i=1}^N f_i M_i$$

Composite

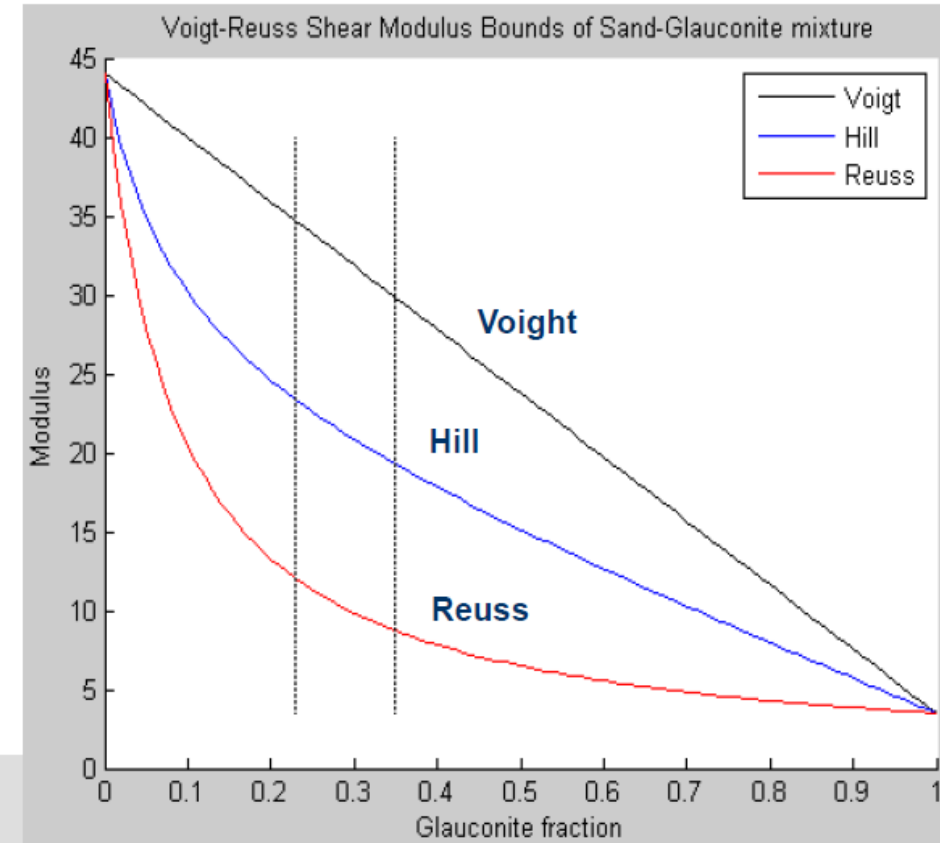


Hill's Elastic Moduli Average (ad-hoc)

$$M_{\text{Solid}} = 0.5(M_V + M_R),$$

$$M_V = \sum_{i=1}^n X_i M_i, \quad M_R = \left(\sum_{i=1}^n \frac{X_i}{M_i} \right)^{-1}$$

M is either bulk or shear modulus; X_i is the volume fraction of i-th mineral in the solid phase; M_i is the corresponding elastic modulus

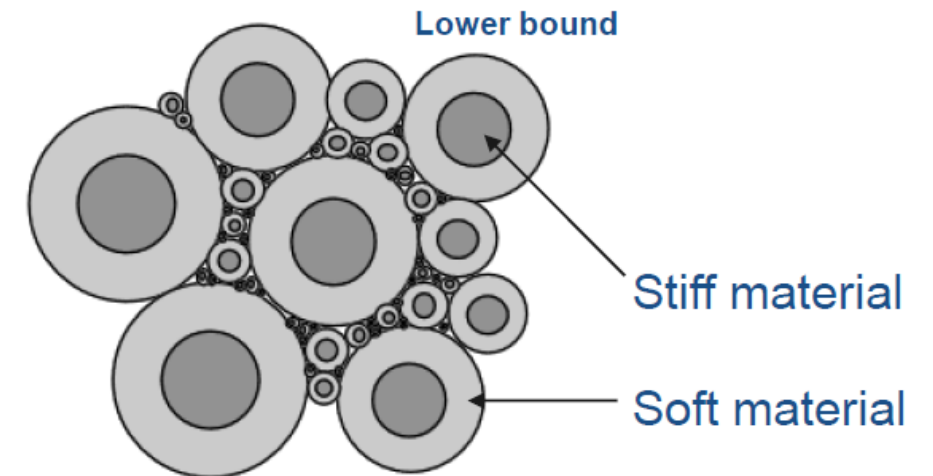
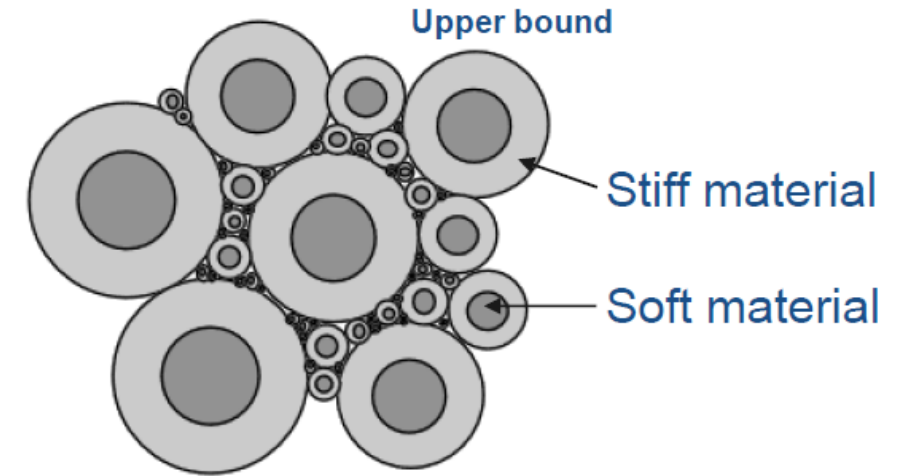


Hashin-Shtrikman Bounds

The Hashin-Shtrikman bounds are equations which define an upper and a lower bound which represent the narrowest range of **velocities possible for a given volume mix of two phases**, either liquid or solid for an isotropic material.

These give upper bounds when the shell is the stiffest material and the lowest bound when the shell is the softer material.

The best bounds for an isotropic elastic mixture, defined as giving the narrowest possible range of elastic moduli.



Hashin-Shtrikman Bounds

For a mixture of two constituent, the Hashin-Shtrikman bounds is given by:

$$K^{HS\pm} = K_1 + \frac{f_2}{(K_2 - K_1)^{-1} + f_1 \left(K_1 + \frac{4}{3} \mu_1 \right)^{-1}}$$

$$\mu^{HS\pm} = \mu_1 + \frac{f_2}{(\mu_2 - \mu_1)^{-1} + \frac{2f_1(K_1 + 2\mu_1)}{5\mu_1 \left(K_1 + \frac{4}{3} \mu_1 \right)}}$$

K_1, K_2 = Bulk moduli of material 1 and 2

μ_1, μ_2 = Shear moduli of material 1 and 2

f_1, f_2 = Volume fractions of material one and 2

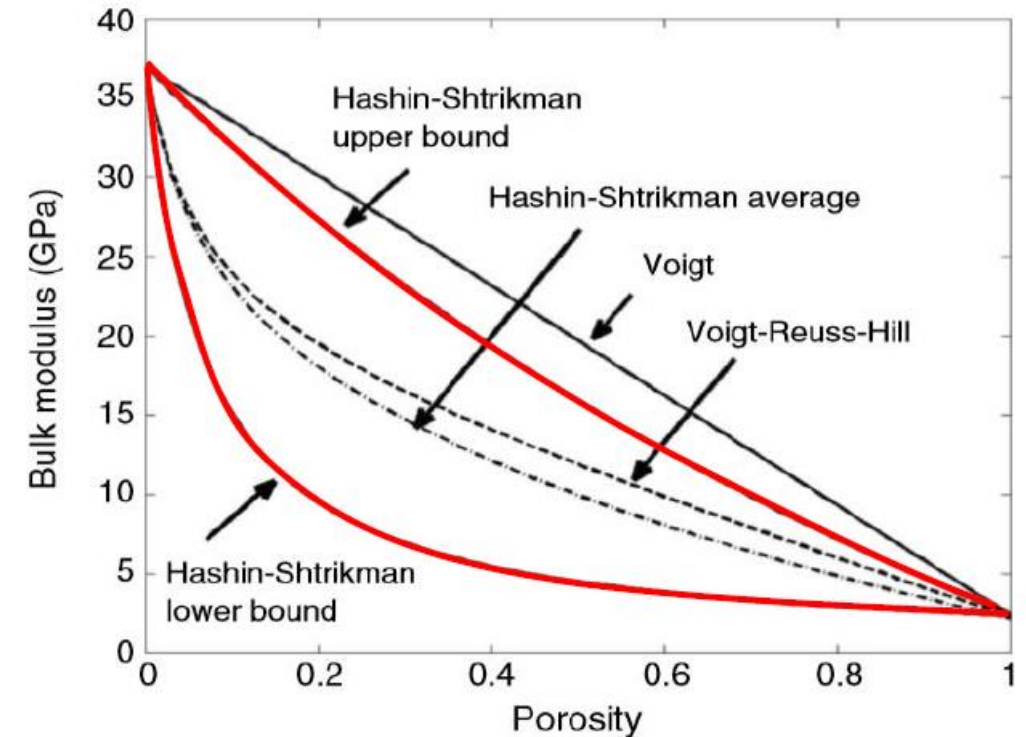
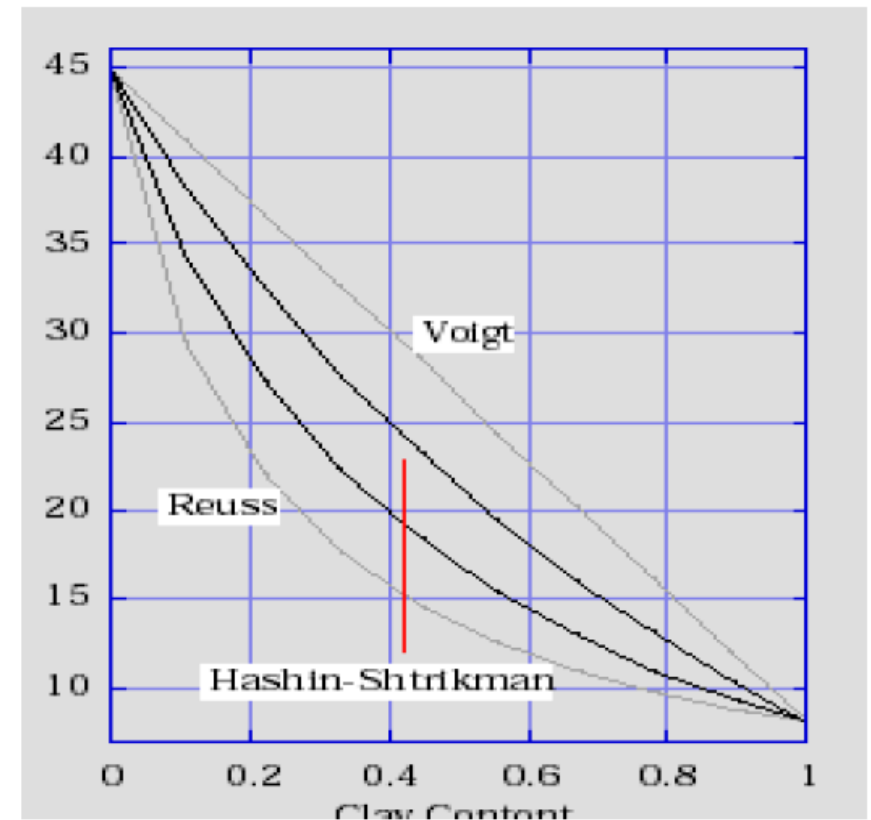


Chart shows mix of quartz mineral
with a brine fluid

Hashin-Shtrikman bounds

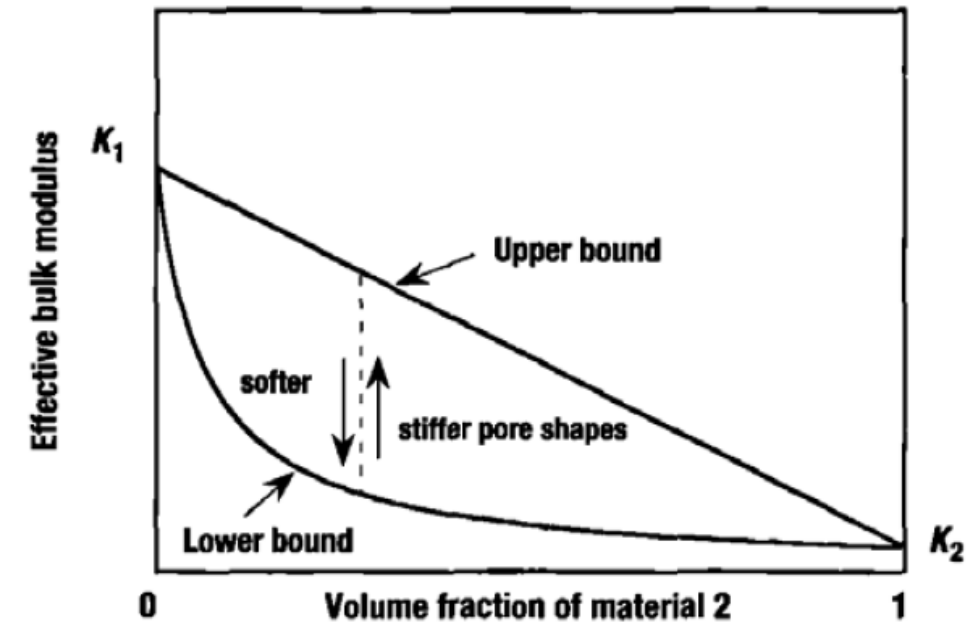
The Hashin-Shtrickman (HS) bounds try to answer what are the tightest possible bounds on effective material properties.

- The only assumption made is that the material is isotropic.
- This means that the microscale and the meso-scale effective properties are both isotropic.



Stiff and soft constituents

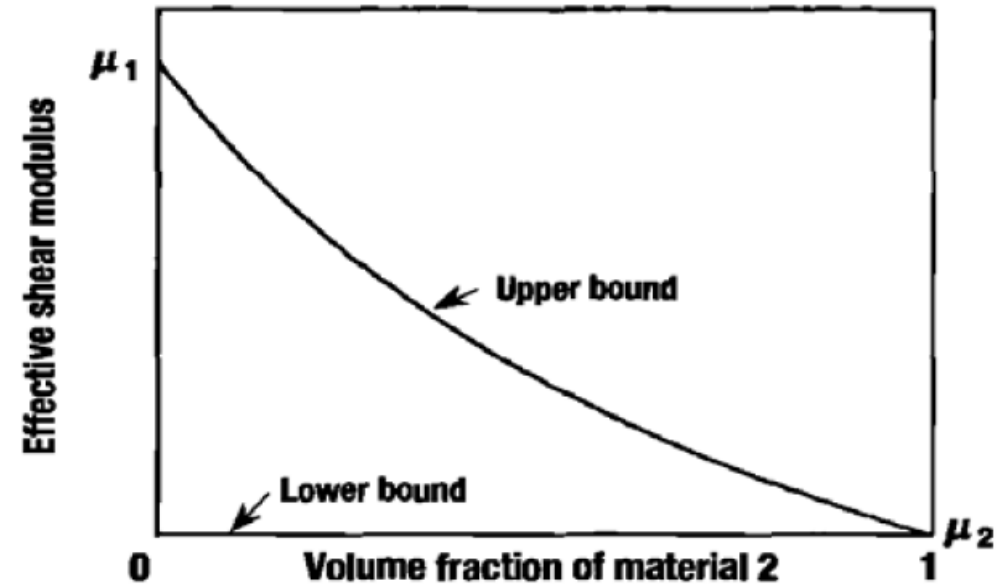
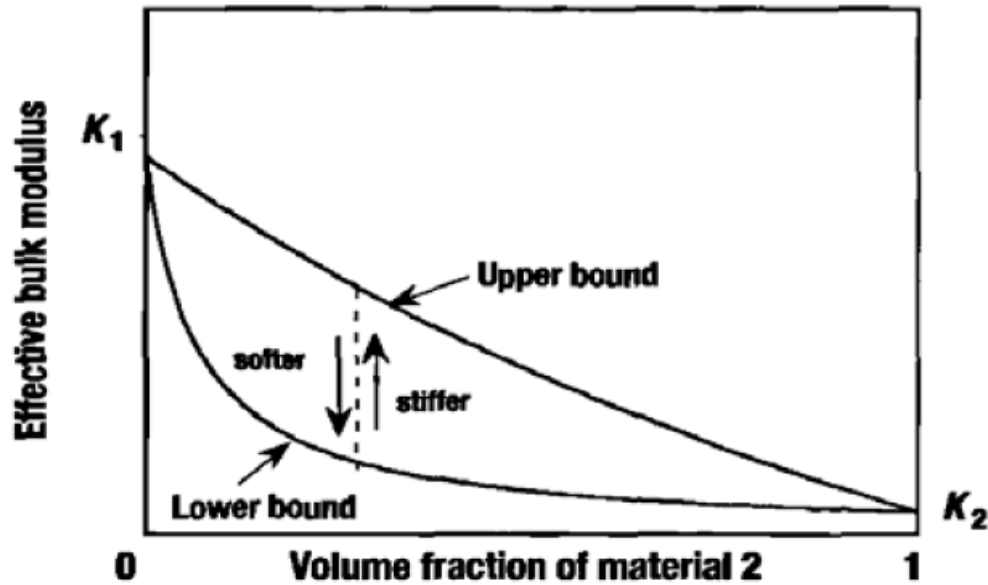
- The mixture constituents might be two different minerals or a mineral plus fluid (water, oil, or gas).
- At any given volume fraction of constituents, the effective modulus of the mixture will fall between the bounds (somewhere along the vertical dash line in the figure), but its precise value depends on the geometric details.
- We use, for example, terms like, “stiff pore shapes” and “soft pore shapes” to describe the geometric variations.
- Stiffer grain or pore shapes cause the value to be higher within the allowable range; softer grain or pore shapes cause the value to be lower.



Conceptional illustration of bounds for the effective elastic bulk modulus of a mixture of two materials

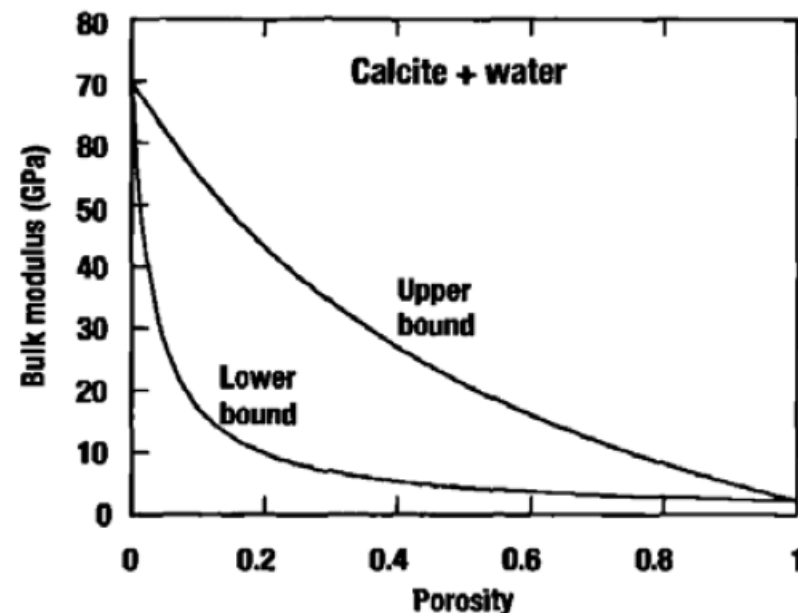
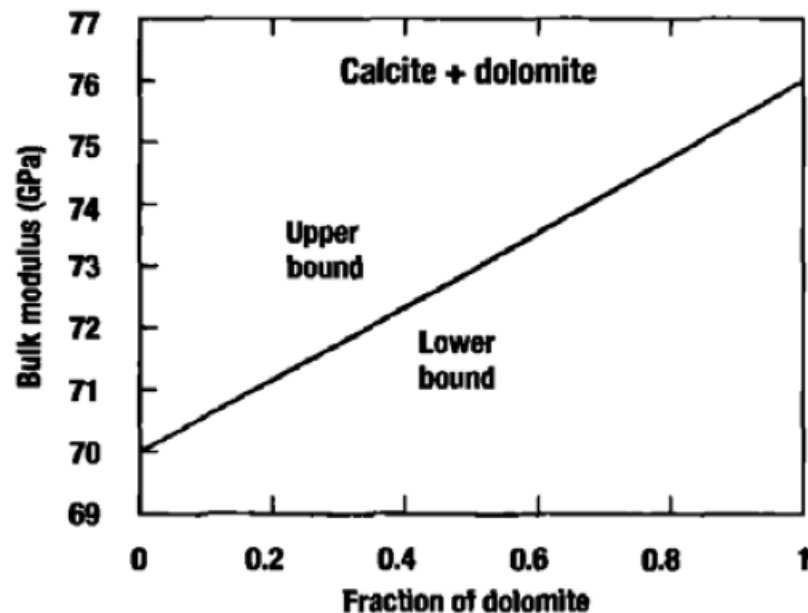
Understanding upper and lower bound

- When one of the constituents is a liquid or gas, the lower bound corresponds to a suspension of the particles in the fluid.
- The lower bound on shear modulus is zero, as long as the volume fraction of fluid is nonzero.



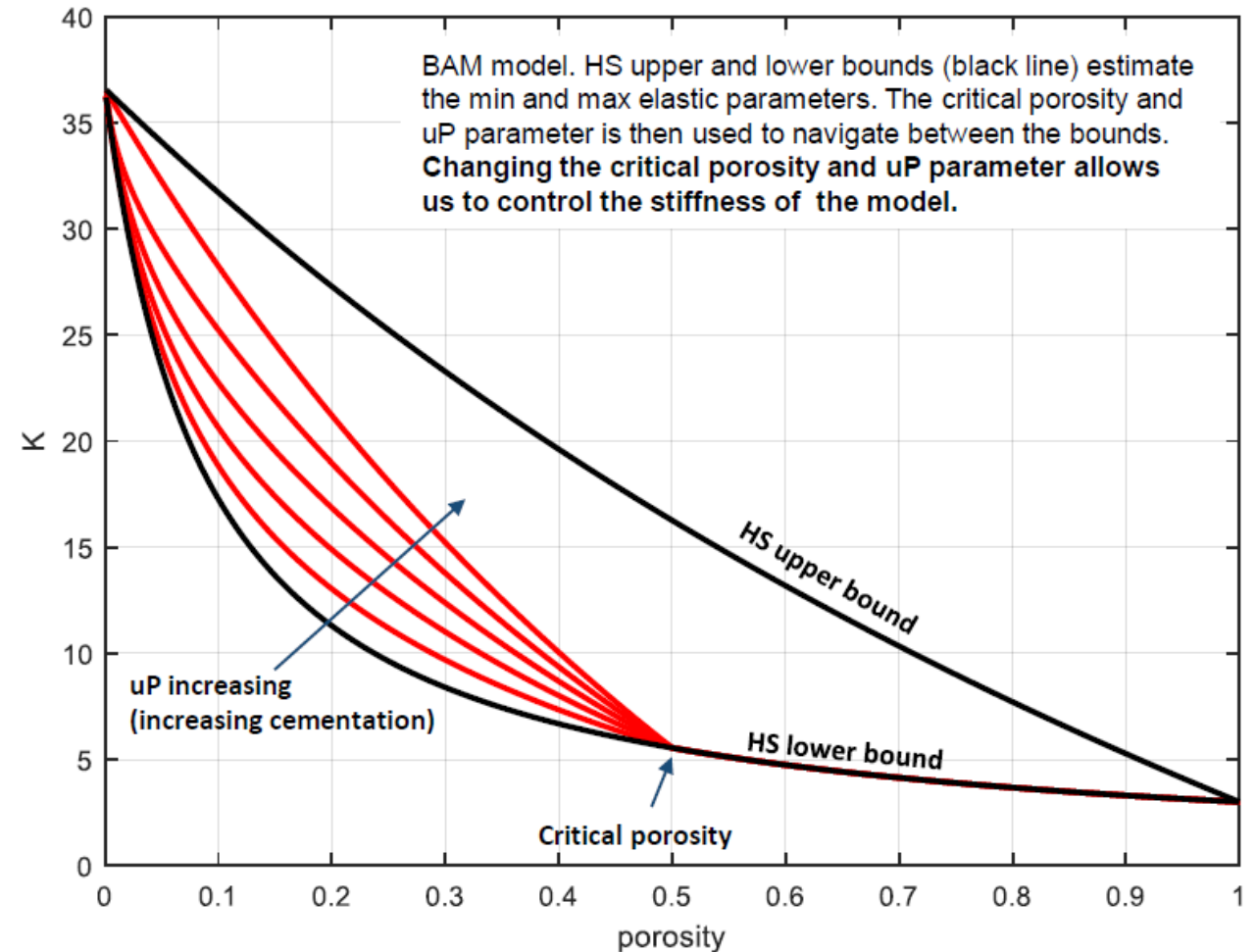
Upper and Lower bound

- The **separation** between the upper and lower bounds (VR or HS) depends on how elastically different the constituents are.
- The bounds are often fairly similar when **mixing solids**, since the elastic moduli of common minerals are usually within a factor of two of each other.
- Many effective medium models (e.g. Gassmann, 1951) assume a **homogenous mineral modulus**, it is often useful (and adequate) to represent a mixed mineralogy with an “average mineral” modulus, equal either to one of the bounds computed for the mix minerals or to their average ($0.5(M_{HSU} + M_{HSL})$)

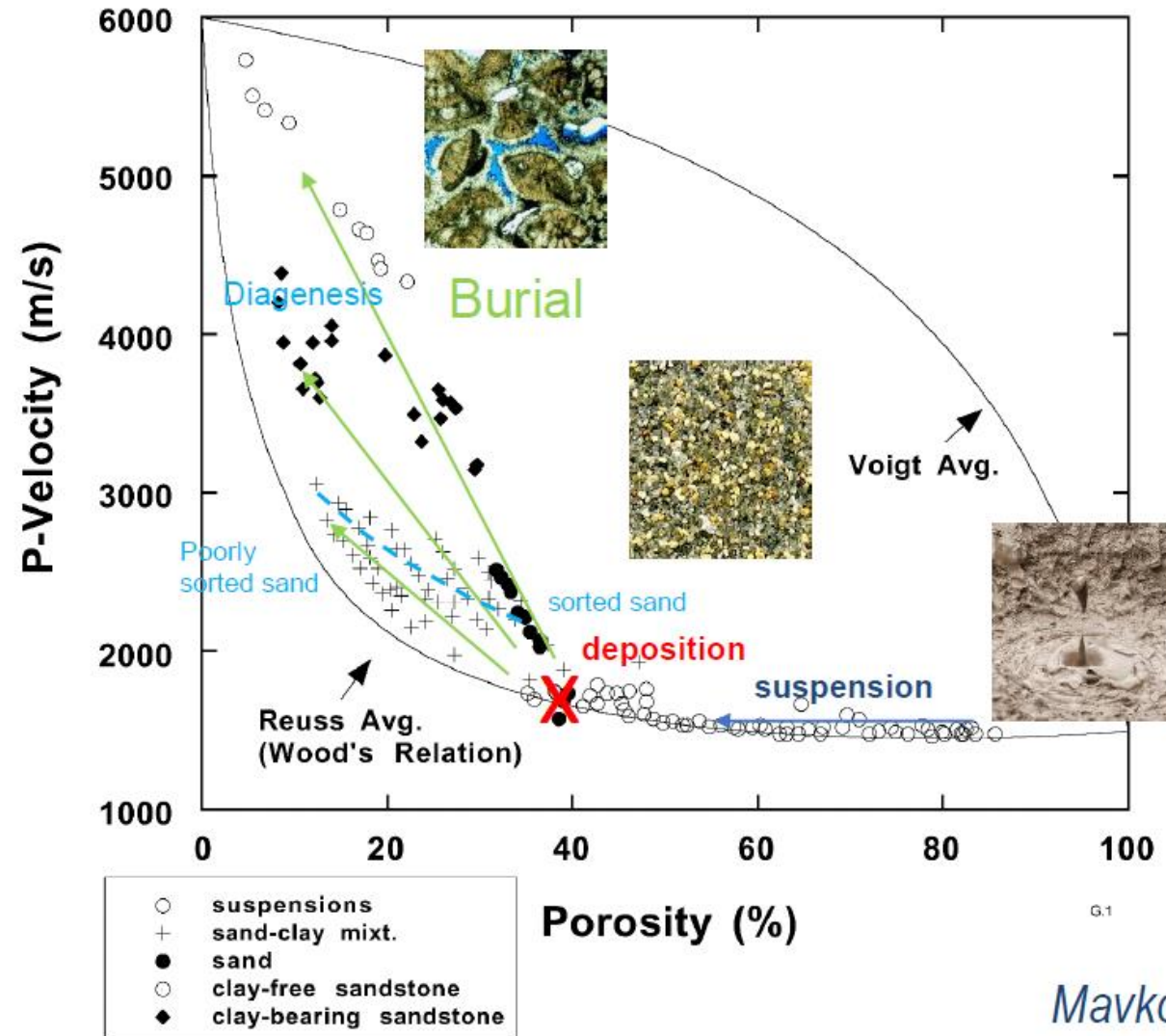


BAM model (Bounding average method)

- The Bounding Average Method (BAM) is a heuristic method based on the theoretical HS upper and lower bounds.
- The HS bounds defined the range of effective moduli for the solid fluid mix. All measured data should fall between the two bounds. Where the data falls between the two is dependent on the microstructure of the rock.
- The fractional vertical position (uP) within the bounds is a measure of the pore geometry (Marion, 1990).



Modelling the rock life cycle



Modelling the rock life cycle

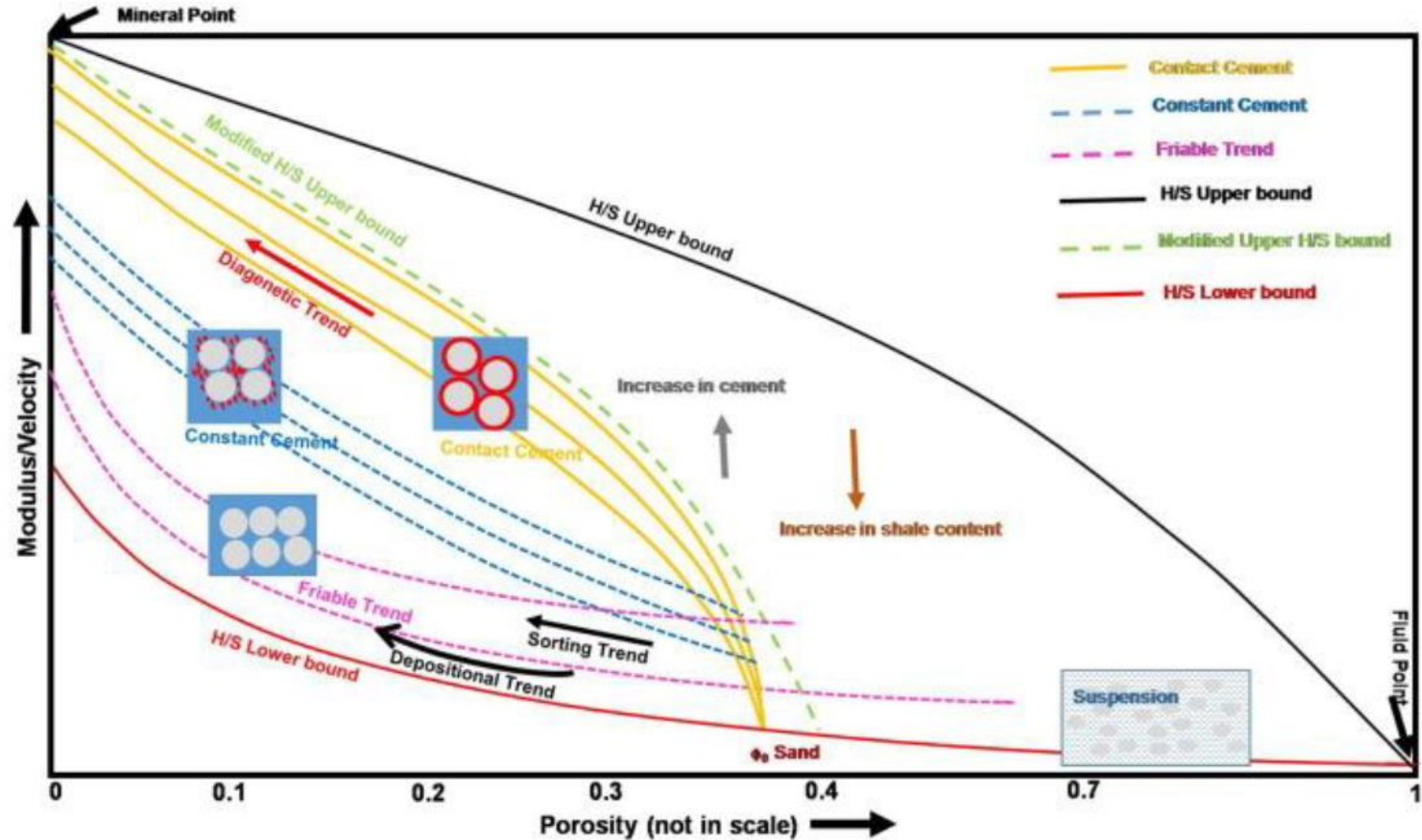


Figure from: Mondal, et al; 2018; Integration of rock physical signatures with depositional environments: A case study from East Coast of India. (After Mavko et al 1998)

Rock model – soft and stiff sand model

The soft sand model (also known as the uncemented sand model) and stiff sand model are granular rock models which calculate the effective properties of a grain pack where the cement is deposited away from the grain contacts.

Hertz Mindlin theory is used to calculate the bulk and shear properties at a critical porosity (~ 0.4) and coordination number. A heuristic modified HS lower bound is then used to estimate the properties at all other porosities. The model is given by

$$K_{HM} = \left[\frac{C^2(1-\varphi_c)^2\mu^2}{18\pi^2(1-\nu)^2} P \right]^{1/3}, \quad G_{HM} = \frac{2+3f-\nu(1+3f)}{5(2-\nu)} \left[\frac{3C^2(1-\varphi_c)^2\mu^2}{2\pi^2(1-\nu)^2} P \right]^{1/3}$$

$$K_{eff} = \left[\frac{\varphi/\varphi_c}{K_{HM} + \frac{4}{3}\mu_{HM}} + \frac{1-\varphi/\varphi_c}{K + \frac{4}{3}\mu_{HM}} \right]^{-1} - \frac{4}{3}\mu_{HM}, \quad \mu_{eff} = \left[\frac{\varphi/\varphi_0}{\mu_{HM} + B} + \frac{1-\varphi/\varphi_c}{\mu + B} \right]^{-1} - B$$

$$\text{Where, } B = \frac{\mu_{HM}}{6} \left(\frac{9K_{HM} + 8\mu_{HM}}{K_{HM} + 2\mu_{HM}} \right)$$

K_{HM} and μ_{HM} are the bulk and shear moduli at critical porosity calculated using Hertz Mindlin theory.

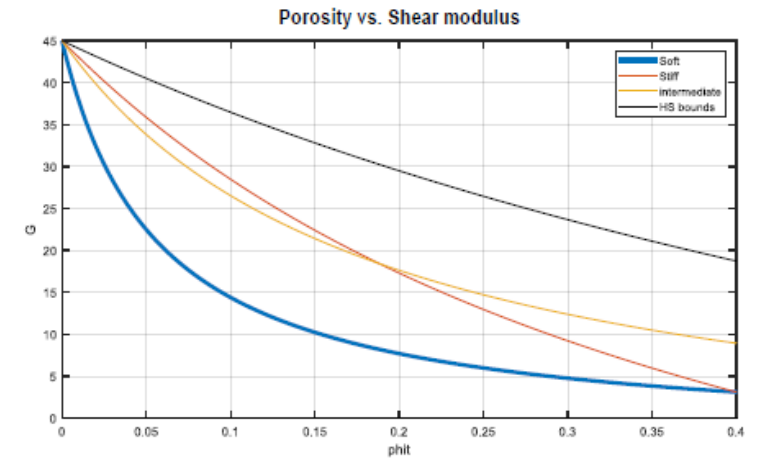
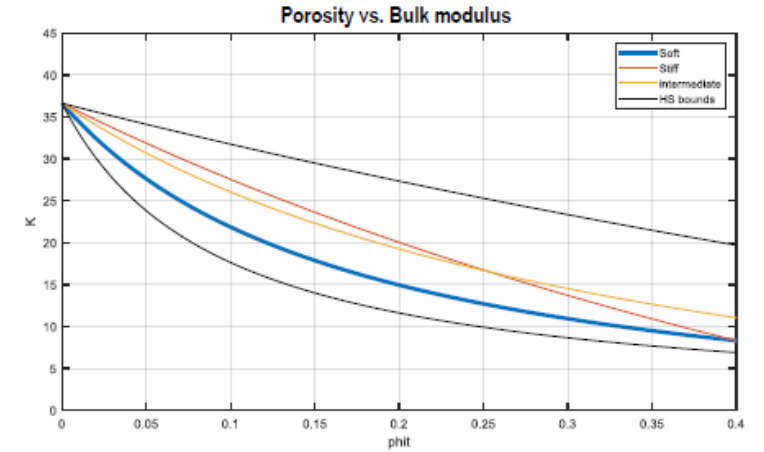
K_{eff} and μ_{eff} are the effective bulk and shear moduli.

f is the fraction of grains with perfect-adhesion.

φ_c is the critical porosity (~ 0.4)

P is the pressure

C is the coordination number

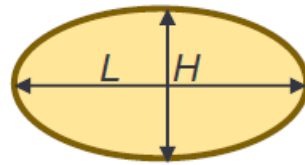


Comparison of the soft sand model to the stiff sand and intermediate sand models. The soft sand model gives much lower moduli estimation as cement is assumed not to stiffen the grain pack.

Inclusion models

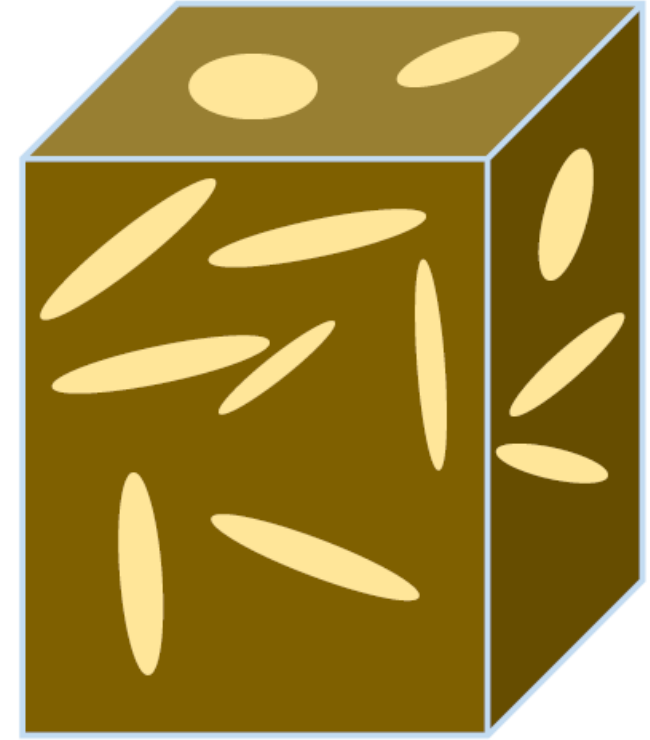
Inclusions models are the theoretical models that approximate the rock as an elastic block of mineral perturbed by holes or describe the rock behaviour by grains in contact.

The Inclusion models describes the pores as inclusions in a background medium. The geometry or “aspect ratio” of the pores defines the stiffness of the dry rock.



Aspect Ratio $\alpha = H/L$

The aspect ratio of 1 equals spherical pores and are stiffest while lower aspect ratio will decrease the stiffness of the pores



Rock model – SCA model

The self consistent approximation (SCA) model is an inclusion type model where the effective properties of the material is based of on deformation of inclusions within a host back material.

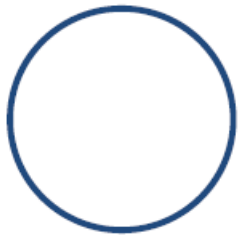
The general self consistent approximation (Berryman 1995) is given by:

$$\sum_{i=1}^N x_i (K_i - K_{SC}^*) P^{*1} = 0$$

$$\sum_{i=1}^N x_i (\mu_i - \mu_{SC}^*) Q^{*1} = 0$$

i refers to the i^{th} material
 SC refers to the final effective medium
 x = volume fraction
 P = geometric factors
 Q = geometric factors
 K = bulk modulus
 μ = shear modulus

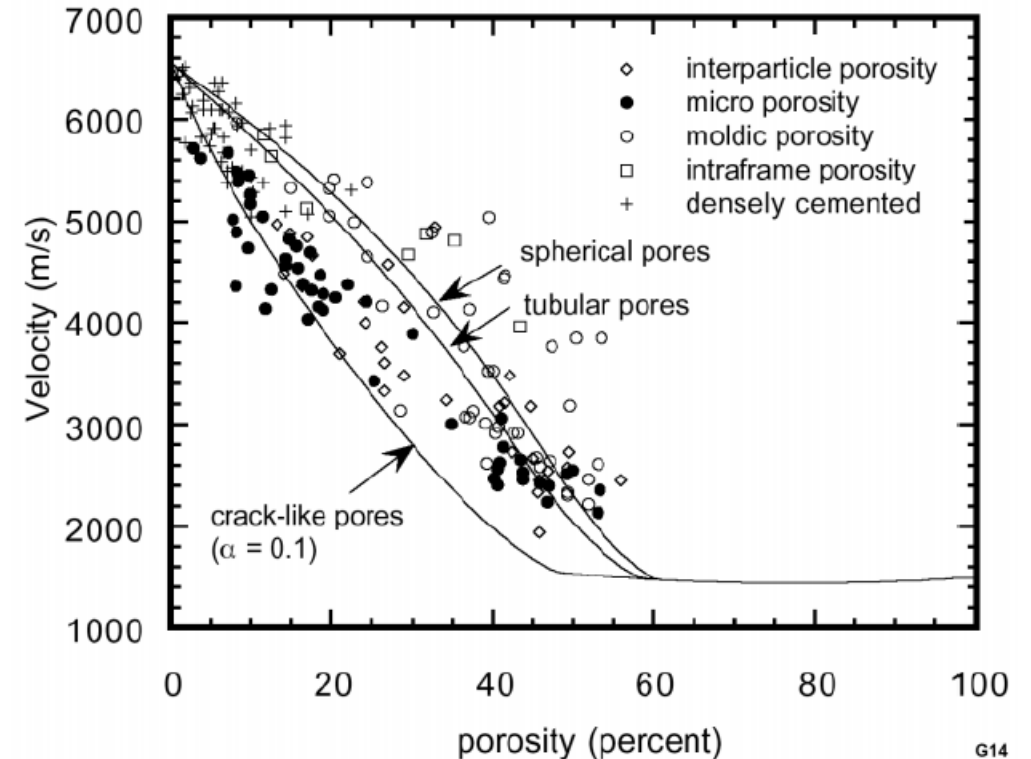
The inclusions are ellipsoidal in shape and the aspect ratio of the inclusions can be altered to control the stiffness of the model (P and Q geometric factors).



Stiff inclusions
 Aspect ratio = 1



Soft inclusions
 Aspect ratio = 0.1



Comparison of SCA model and carbonate data
 (figure from Stanford rock physics course (Gary Mavko) – data from Anselmetti and Eberli., 1997)

Rock model – DEM model

It mixes two phases (matrix and pores) by considering pore shape and by applying the analytical proposed by Eshelby's (1957) to estimate the differential increment in stiffness from the small amount of phase 2 (pores) introduced into phase 1 (host material)

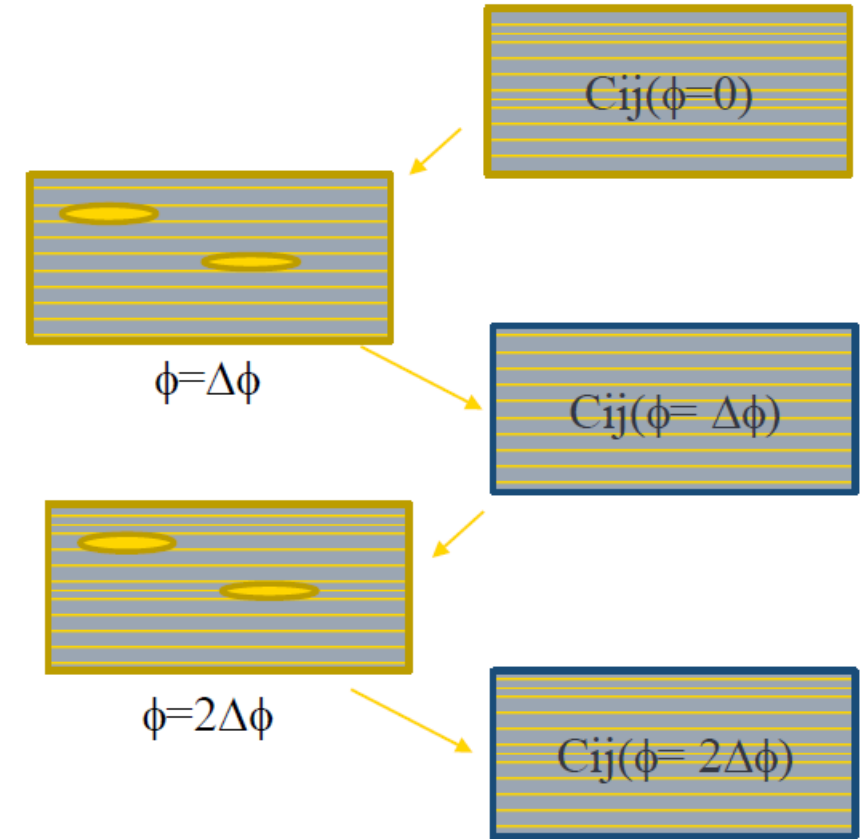
$$dC^{DEM}(\nu, \alpha) = \frac{d\nu_i}{1 - \nu_i} (C^i - C^{DEM}(\nu, \alpha)) Q_i(\alpha)$$

$$C^{DEM}(\phi, \alpha) = \int_{\nu=0}^{\phi} dC^{DEM}(\nu, \alpha, C^{DEM}(\nu, \alpha))$$

Hornby et al, 1994

Anisotropic formulation: anisotropic background matrix and ellipsoidal pores with given aspect ratio

Critical porosity is a useful constrain to ensure the host material does not exceed porosity larger than depositional porosity



After Zimmerman, 1991

Rock model – DEM model

It mixes two phases (matrix and pores) by considering pore shape and by applying the analytical proposed by Eshelby's (1957) to estimate the differential increment in stiffness from the small amount of phase 2 (pores) introduced into phase 1 (host material)

$$dC^{DEM}(v, \alpha) = \frac{dv_i}{1 - v_i} (C^i - C^{DEM}(v, \alpha)) Q_i(\alpha)$$

$$C^{DEM}(\phi, \alpha) = \int_{v=0}^{\phi} dC^{DEM}(v, \alpha, C^{DEM}(v, \alpha))$$

Hornby et al, 1994

Other Inclusion models

- Self-Consistent approximation of effective moduli.
- Hudson model
- Cheng effective moduli
- Xu-White

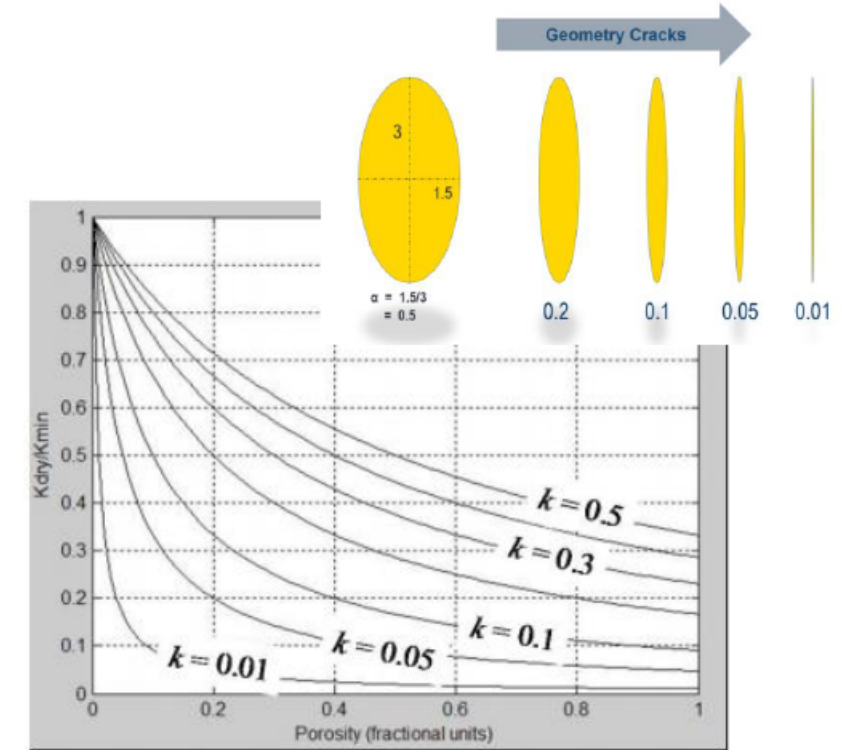


FIG. 1. A family of dry rock over matrix bulk modulus ratio curves for varying values of k . This type of plot can be used for pore-shape determination

Determining Pore Type/Aspect Ratio Classes

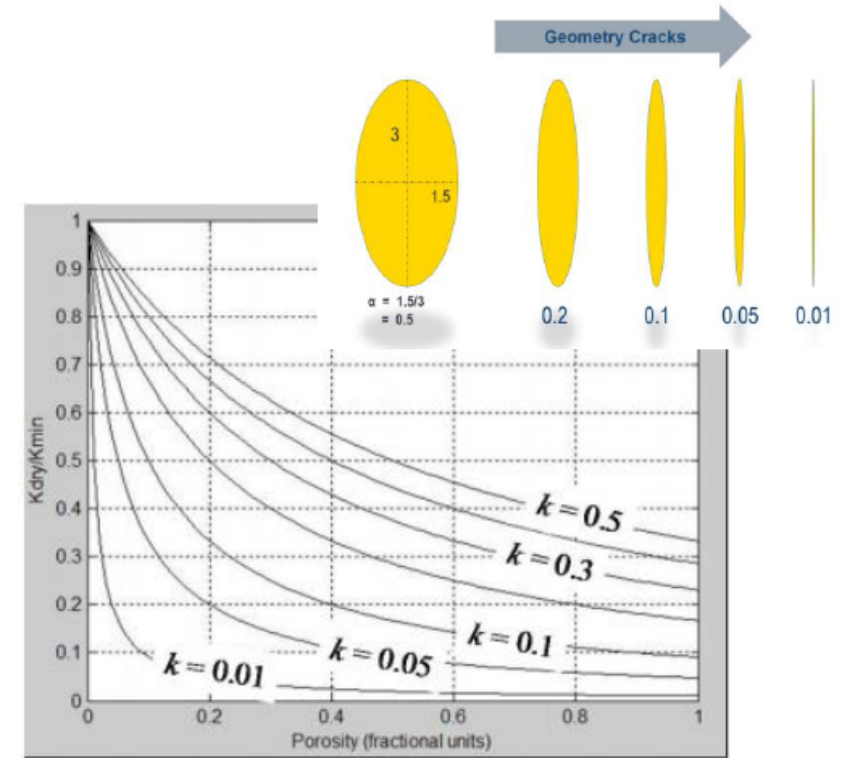
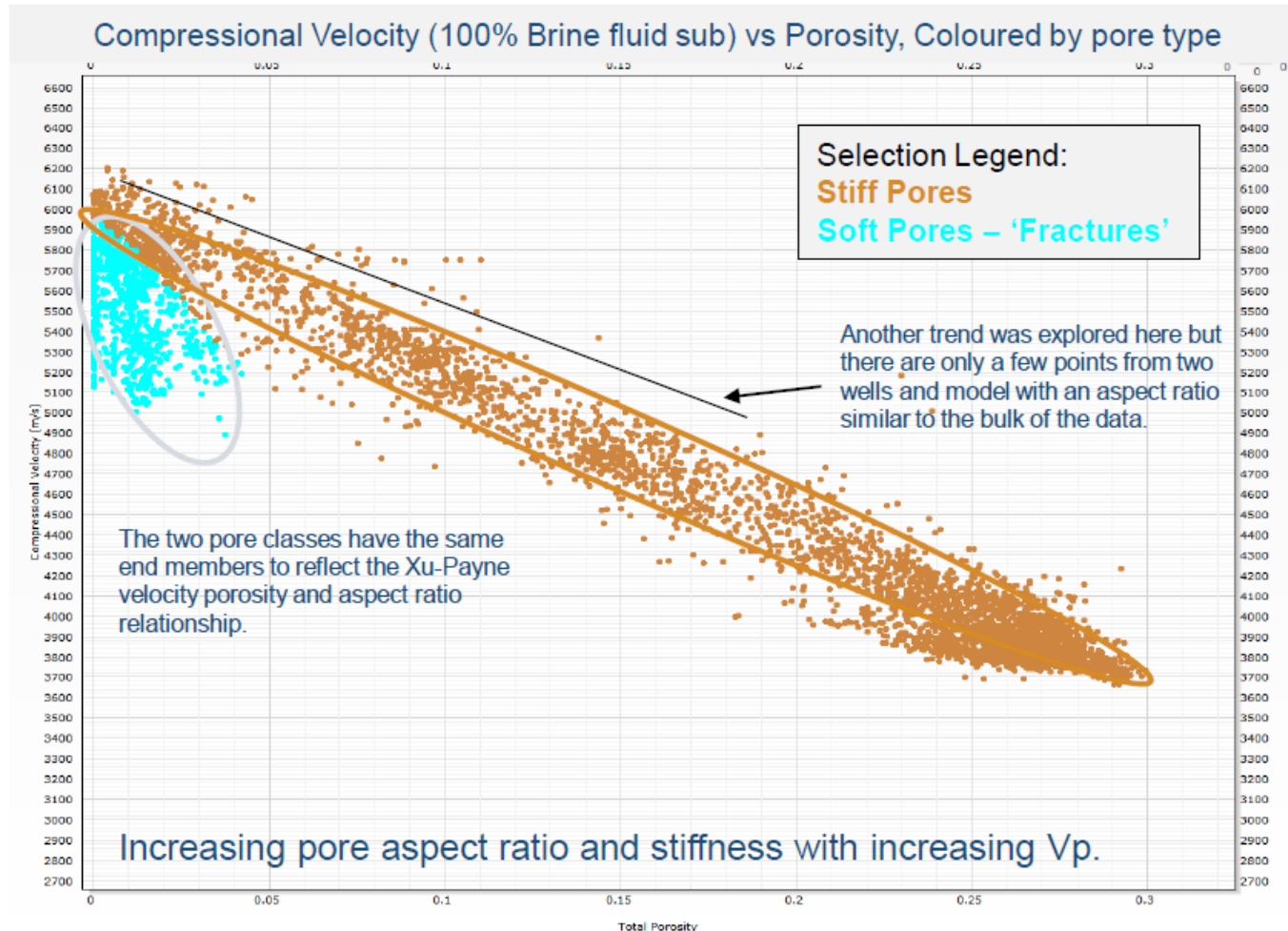
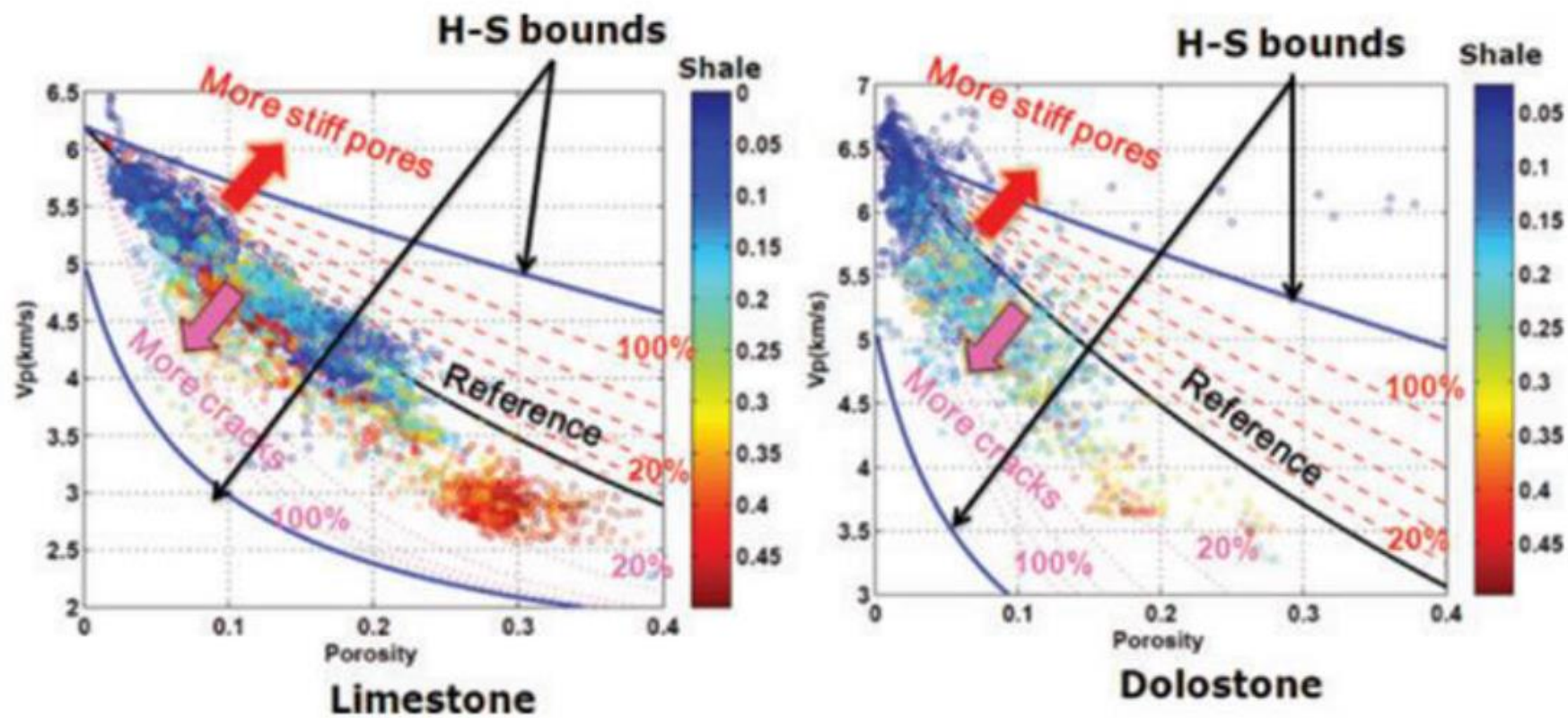
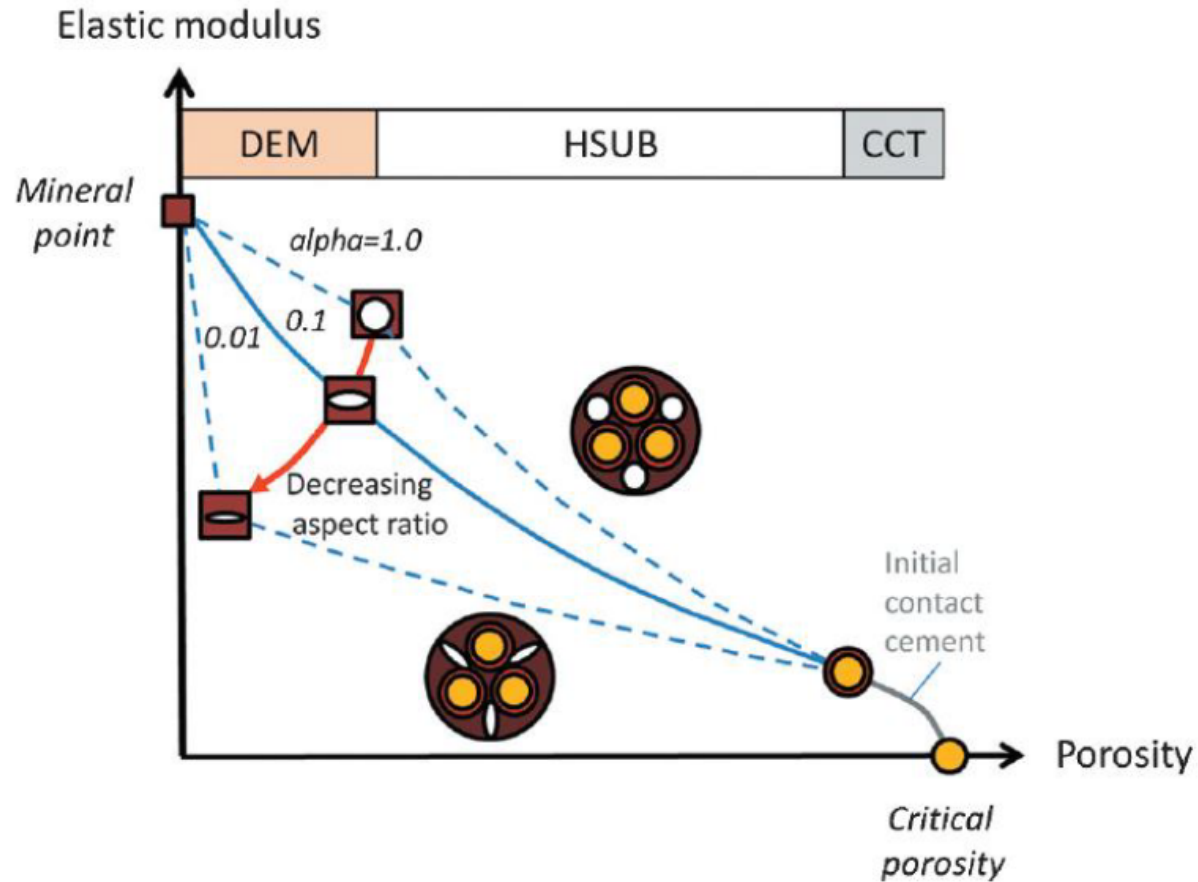


FIG. 1. A family of dry rock over matrix bulk modulus ratio curves for varying values of k . This type of plot can be used for pore-shape determination

Example of calibrated rock model



Combining models – Kite model



3 stage model (Avseth et al. 2014)

This rock model describes the elastic properties of low to intermediate porosity rocks with a range of rock textures

Step 1. Use DEM model for the low porosity rocks

Step 2. Use CCT model for the high porosity rock at the cementation point

Step 3. Use the HS bound to compute the elastic properties at the intermediate porosities

Refer to *Rock-physics modeling guided by depositional and burial history in low-to-intermediate-porosity sandstones*

Per Avseth, Tor Arne Johansen, Aiman Bakhorji and Husam M. Mustafa,

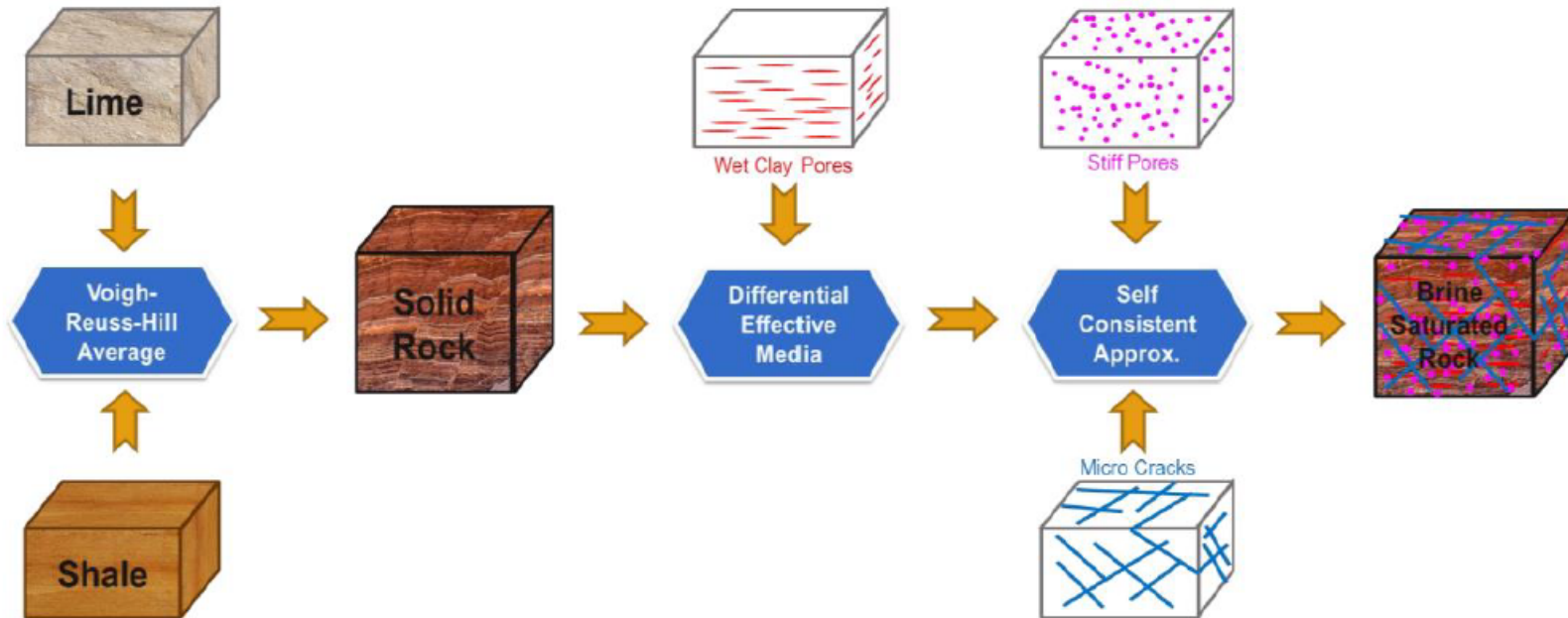
Combining models

Schematic representation of modelling workflow used to reproduce the complexity of a carbonate reservoir rock (described by Zhao '13)

In carbonate rocks, porosity and permeability are not simply related. For the same porosity it is possible to have multiple permeability clusters based on pore shapes.

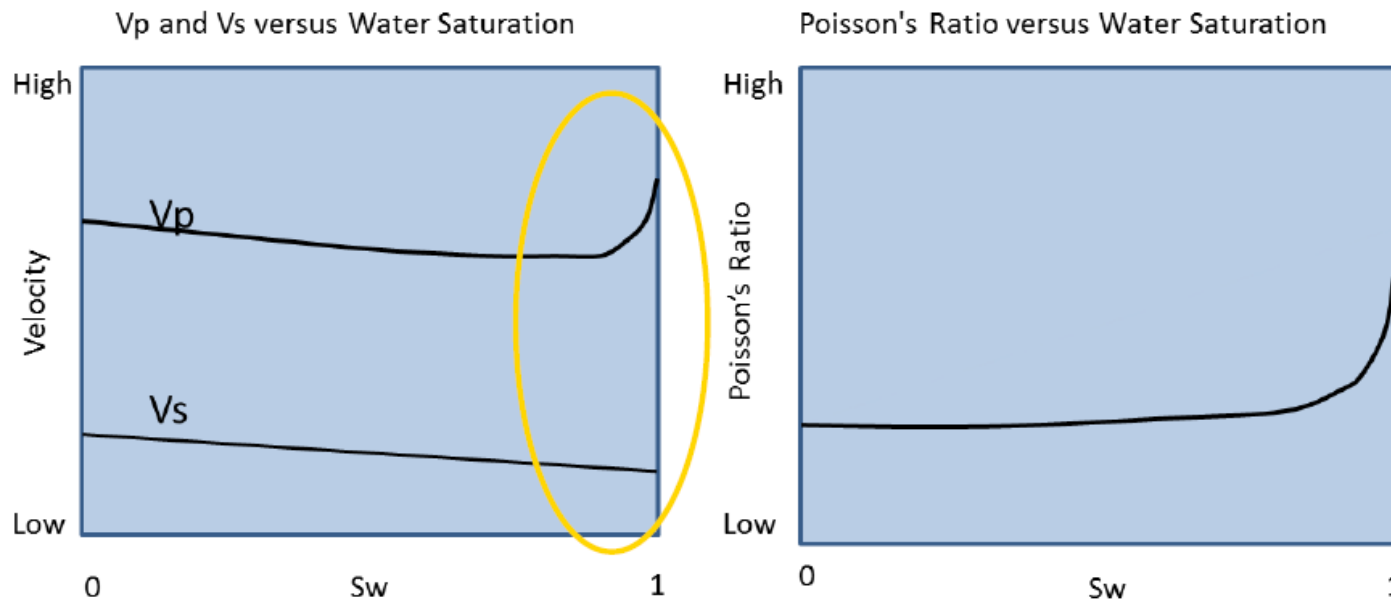
Typically to map facies for carbonate rocks it is important to capture the poro-perm based facies classes or the reservoir rock types.

In a particular field most of the production can be driven by vuggy or moldic pores hence it is important to understand through rock physics modelling and analysis how the stiffer pores separate out from the cracks or softer ones in the elastic parameter domain.



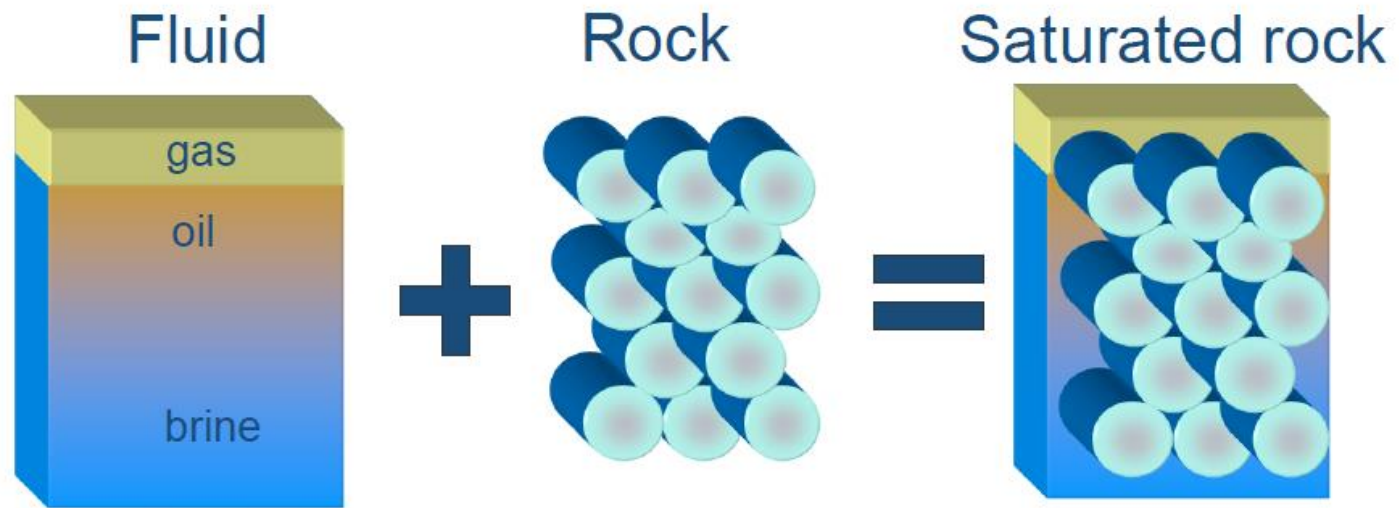
Saturation effects

Using Gassmann's formula, we can investigate the effect of the introduction of oil/gas on P wave and S wave velocities

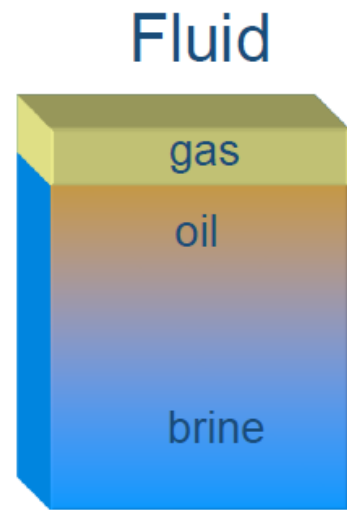


Gas saturation effect in a porous uncompact rock is good to differentiate fluid from lithology change

Introduction – Fluid properties



Introduction – Fluid properties



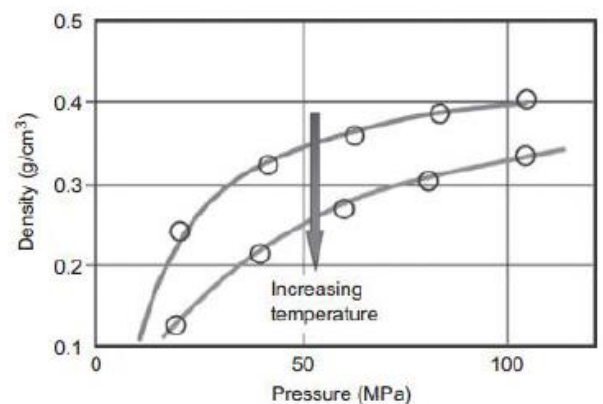
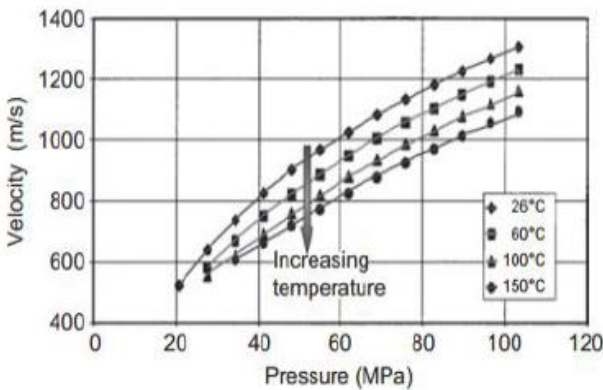
We want to know the density and velocity (or bulk modulus) of the fluids in the system.

Batzle & Wang

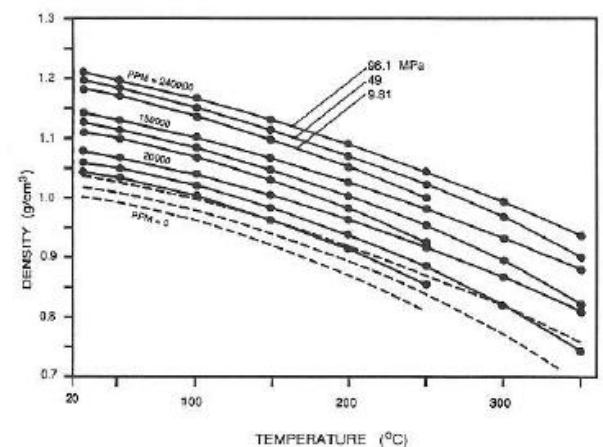
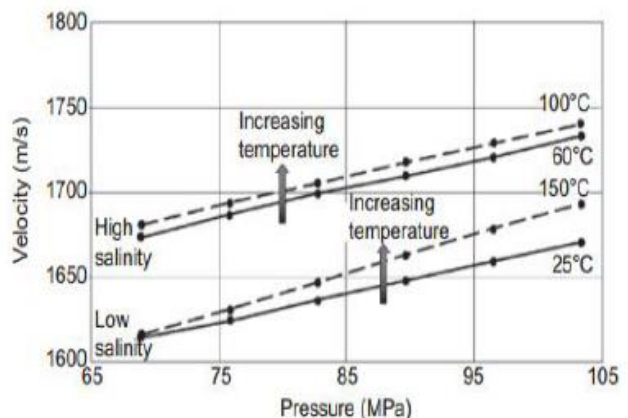
- Batzle and Wang (1992) combined thermodynamic relationships and empirical trends from published data to predict the effects of pressure, temperature and composition on the seismic properties of fluids.
- These empirical relations calculate insitu density and velocity (or modulus).
- In addition to temperature and pressure the inputs for the different phases are:
 - Water: salinity in ppm (can be calculated from R_w), dissolved gas
 - Oil: density at standard conditions in API, and solution GOR
 - Gas: relative gas gravity

Fluid Velocities and Densities with Pressure & Temperature

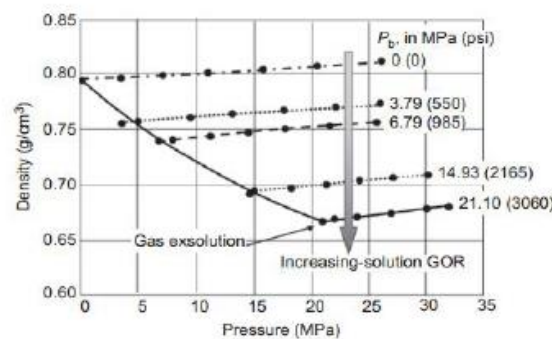
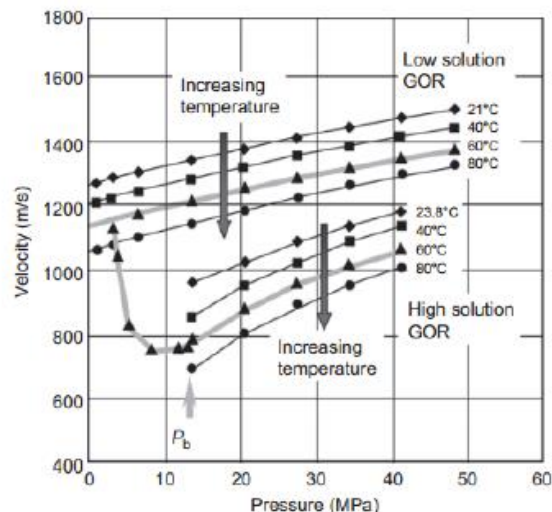
Gas



Brine



Oil



Batzle & Wang

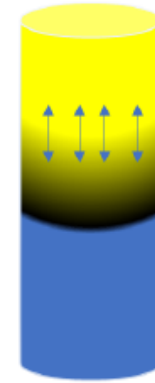
- Batzle and Wang (1992) equations are valid for a Black Oil system.
- Examples where Batzle and Wang (1992) equations are not valid:
 - Heavy oils
 - Gas condensates where there is a phase transition between the fluids.

Black Oil

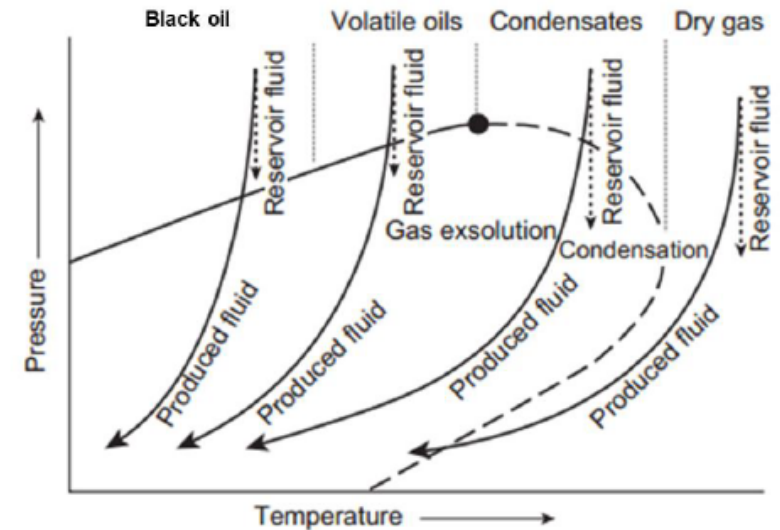


B&W (1992) Assumes composition of fluid phases constant at all T and P.

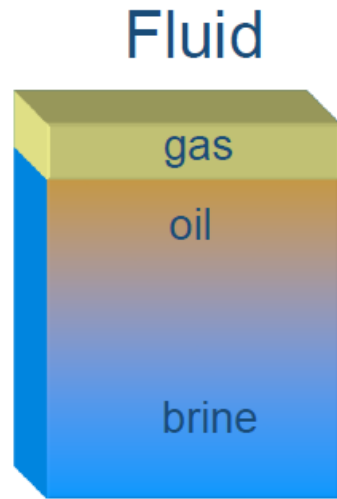
Compositional and Thermal



EOS represents fluids at all T, P, Composition.



Introduction – Fluid properties and mixing



We want to know the density and velocity (or bulk modulus) of the fluids in the system.

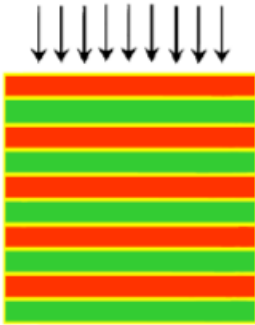
We want to perform fluid substitution either to or to a partial saturation (gas/water) or mixed saturation (gas/water/oil).

The most common method is to replace the collection of phases with a single “effective fluid”.

This is done through fluid mixing.

Fluid mixing

Isostress average

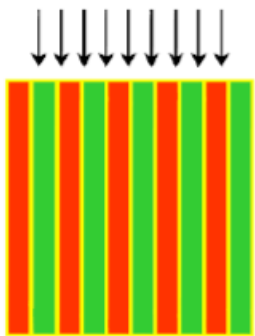


Reuss lower bound
Homogeneous mix
 Harmonic mean

$$\frac{1}{K_{eff}} = \frac{S_w}{K_w} + \frac{S_o}{K_o}$$

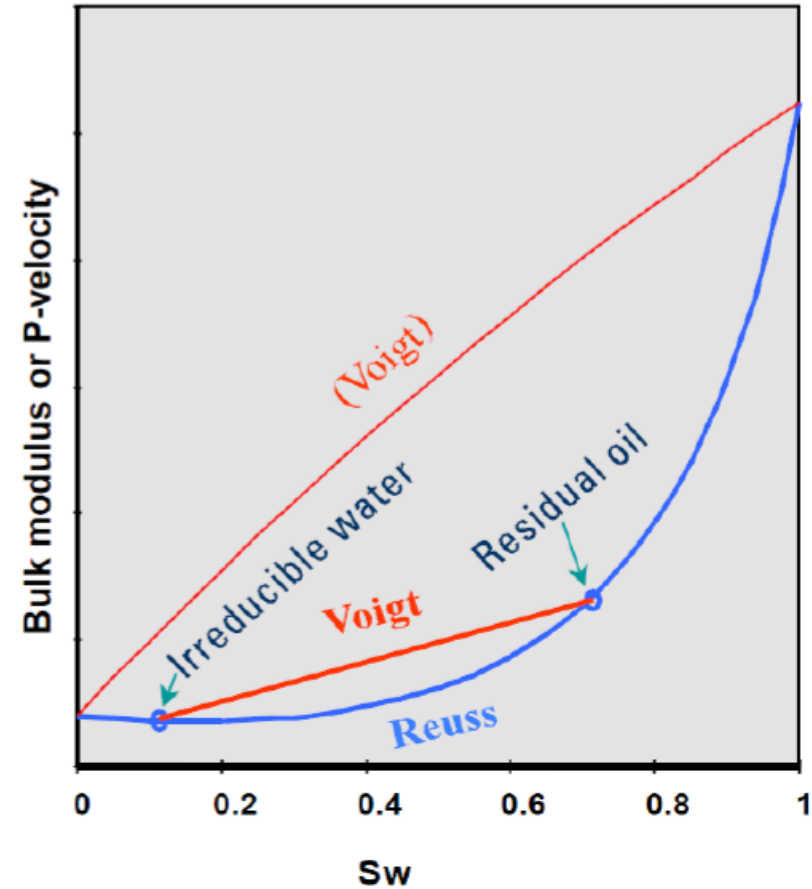
Wood's equation

Isostrain average



Voigt upper bound
Patchy mix
 Arithmetic mean

$$K_{eff} = S_w K_w + S_o K_o$$



Understanding fluids mixing

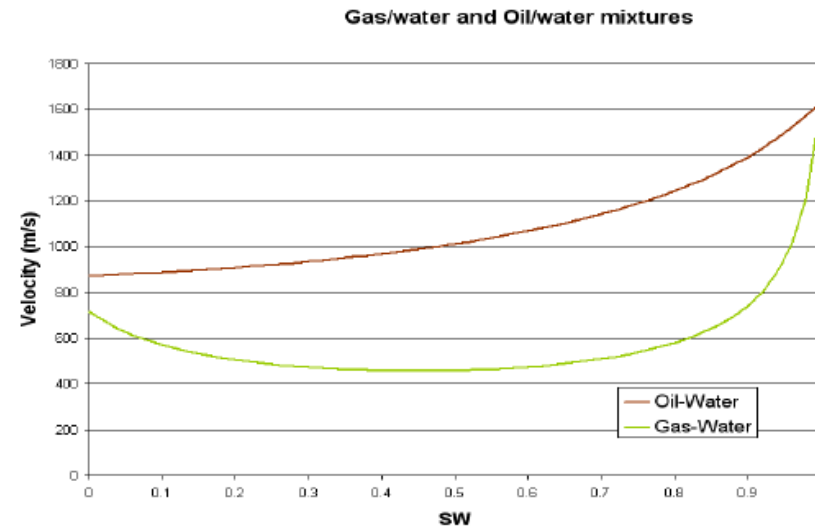
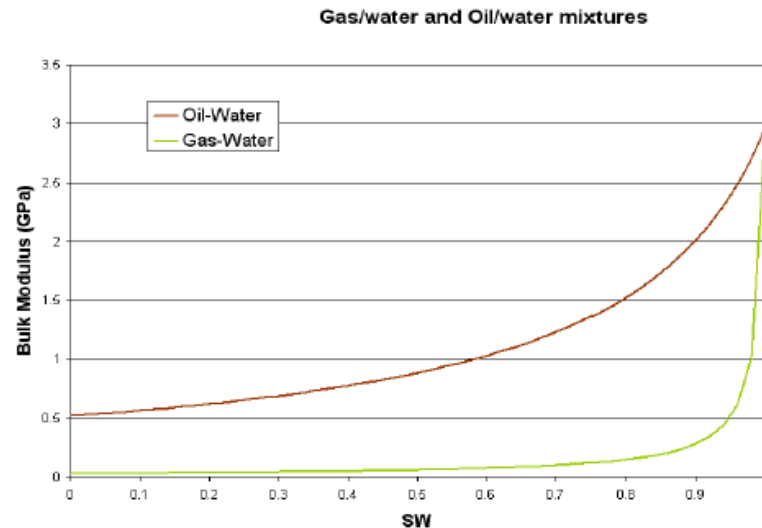
- The Reuss average assumes pressure of the two phases are equal. In other words, the individual fluid phases are in perfect hydraulic communication
- Reuss average is closer to the corresponding signature in seismic response

$$\frac{1}{K_f} = \sum \frac{S_i}{K_i}$$

K_f – fluid mixture bulk modulus

K_i bulk moduli of individual fluids

S_i = saturations of individual fluids



Patchy Mixing

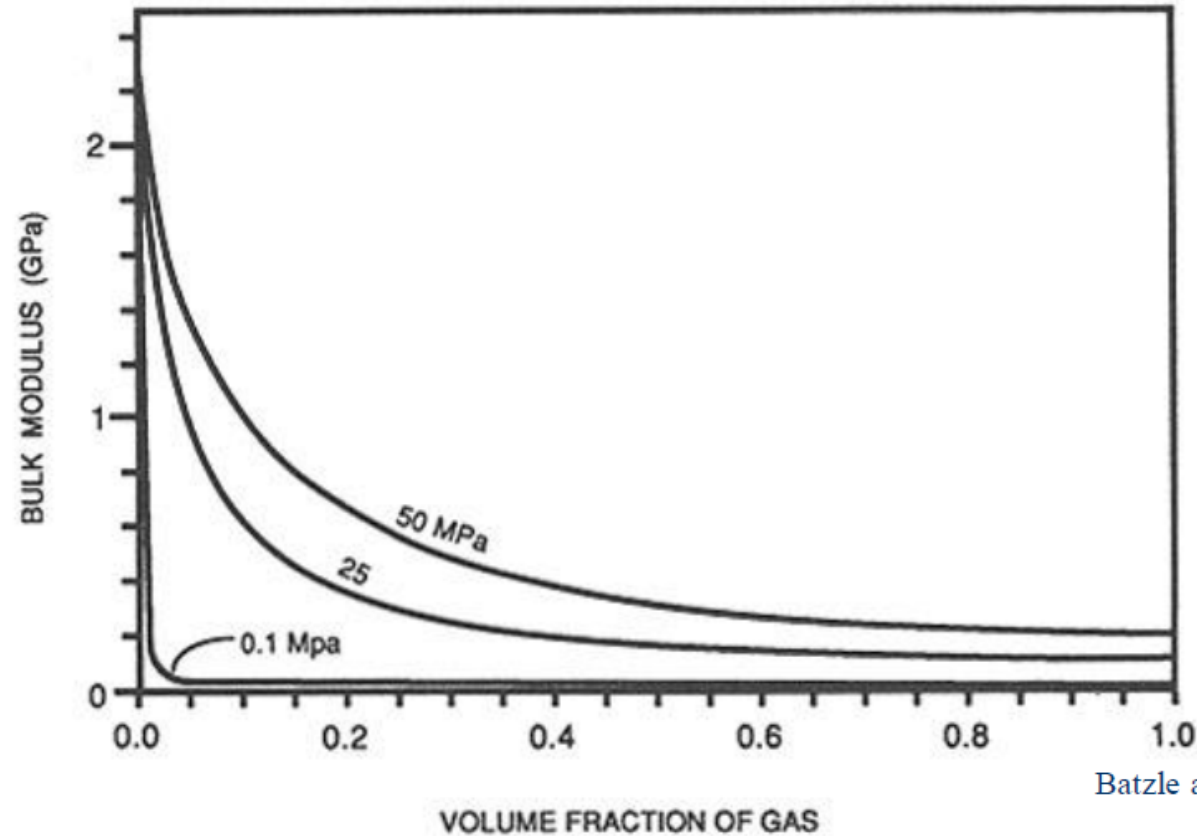
Patchy saturation describes the situation when the saturation is spatially variable so that each “patch” have fluid phase equilibrium but is not in equilibrium with neighbouring “patches”.

The Voigt average can be used as
an approximation to patchy upper bound:

$$K_{fl} = \sum S_i K_i$$

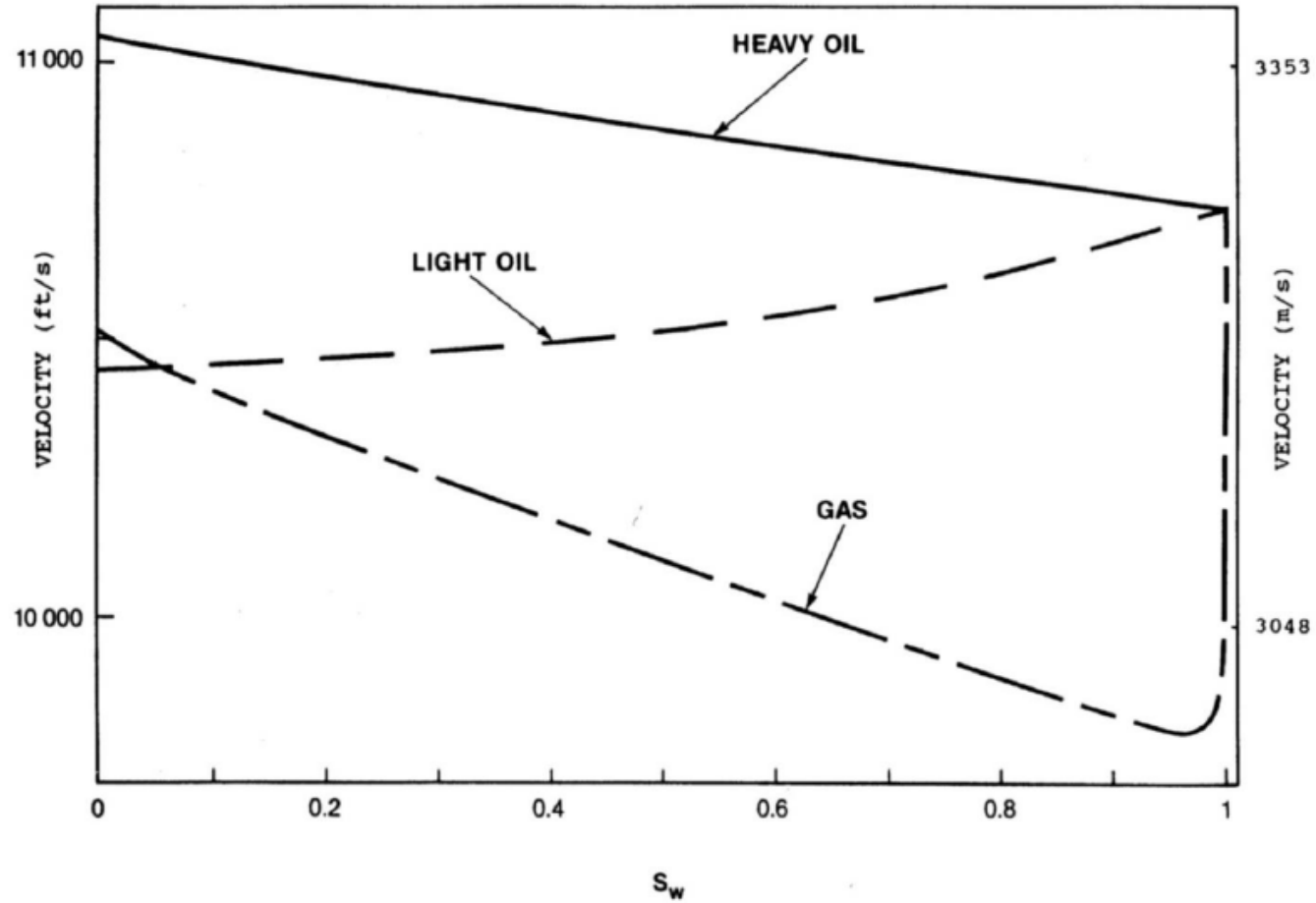
Fluids – mixtures of phases

Bulk modulus of free gas-brine mix



The calculated bulk modulus for mixtures of gas ($G = 0.6$) and brine (50 000 ppm NaCl). The approximate in-situ temperatures were used at each pressure (0.1 MPa-20°C; 25 MPa-68°C; 50 MPa-116°C).

Example of fluid effect on velocity



Fluids in rocks

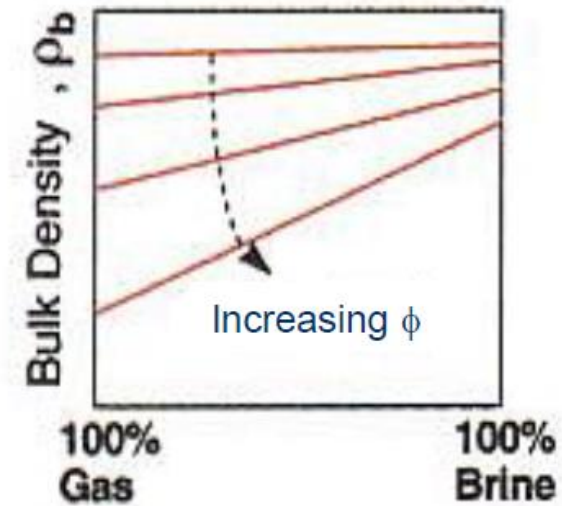


Grain/matrix

Pore fluid

Porosity, pore fluid volume fraction ϕ Grain/matrix

Higher porosity means greater impact of the fluid composition



Gassmann (1951)

$$V_p = \sqrt{\frac{K_{sat} + 4\mu/3}{\rho}} \quad ; \quad \mu_{sat} = \mu_{dry}$$

$$K_{sat} = \underbrace{K_{dry}}_{\text{Total rock}} + \underbrace{\frac{(1 - K_{dry}/K_m)^2}{\Phi/K_{fl} + (1-\Phi)/K_m - K_{dry}/K_m^2}}_{\text{Dry frame Pore system + pore fluid}}$$

Works best for very low frequencies in situ seismic data (<100Hz) and may perform less well as frequencies increase towards sonic logging ($\approx 10^4$ Hz) and laboratory measurements ($\approx 10^6$ Hz)

Gassmann fluid substitution (1951)

$$\frac{K_1}{K_0 - K_1} - \frac{K_{fl1}}{\varphi(K_0 - K_{fl1})} = \frac{K_2}{K_0 - K_2} - \frac{K_{fl2}}{\varphi(K_0 - K_{fl2})}$$

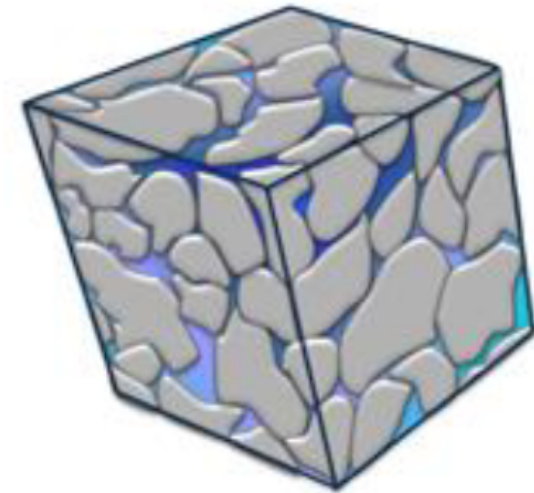
$$\mu_{sat} = \mu_{dry}$$

The equations can be rewritten to directly substitute the rock from containing one fluid to another.

Gassmann assumptions

These key assumptions in Gassmann's equation:

- The rock is elastically isotropic and homogeneous.
- The pore space is in perfect communication.
- The pore fluid is frictionless and there is no change in properties of solid due to fluid saturation.
- The rock and fluids can be characterized by single bulk moduli and densities.



Fluid substitution steps

1. Use the formula for the shear velocity to calculate μ $\mu = \rho V_s^2$

2. Calculate K_{sat} $V_p = \sqrt{\frac{K_{sat} + \frac{4}{3}\mu}{\rho}}$

3. Calculate K_{dry} from K_{sat}

$$K_{sat} = \left(K_{dry} + \frac{\left(1 - \frac{K_{dry}}{K_m}\right)^2}{\frac{\phi}{K_{fl}} + \frac{1 - \phi}{K_m} + \frac{K_{dry}}{K_m^2}} \right)$$

4. Use published values for K_m , K_{fluid1}

5. Recalculate K_{sat} for a new fluid with K_{fluid2}

6. Calculate the new bulk density $\rho_b = \rho_m(1 - \phi) + \rho_w S_w \phi + \rho_{HC}(1 - S_w)\phi$

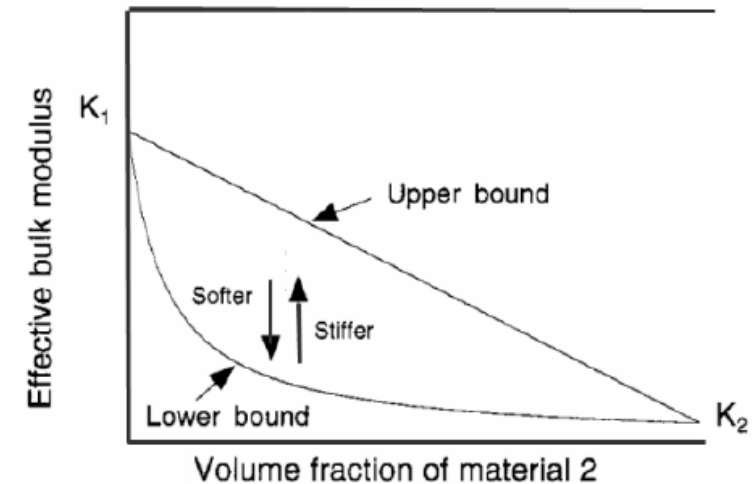
7. Calculate the new V_p

Mixing formulas

Gassmann's equation assumes that the fluids and the rock frame are represented by single values for the densities and the bulk moduli

For mineral mixtures (Quartz and Clay) this is the same. They are represented by one value for density and bulk moduli

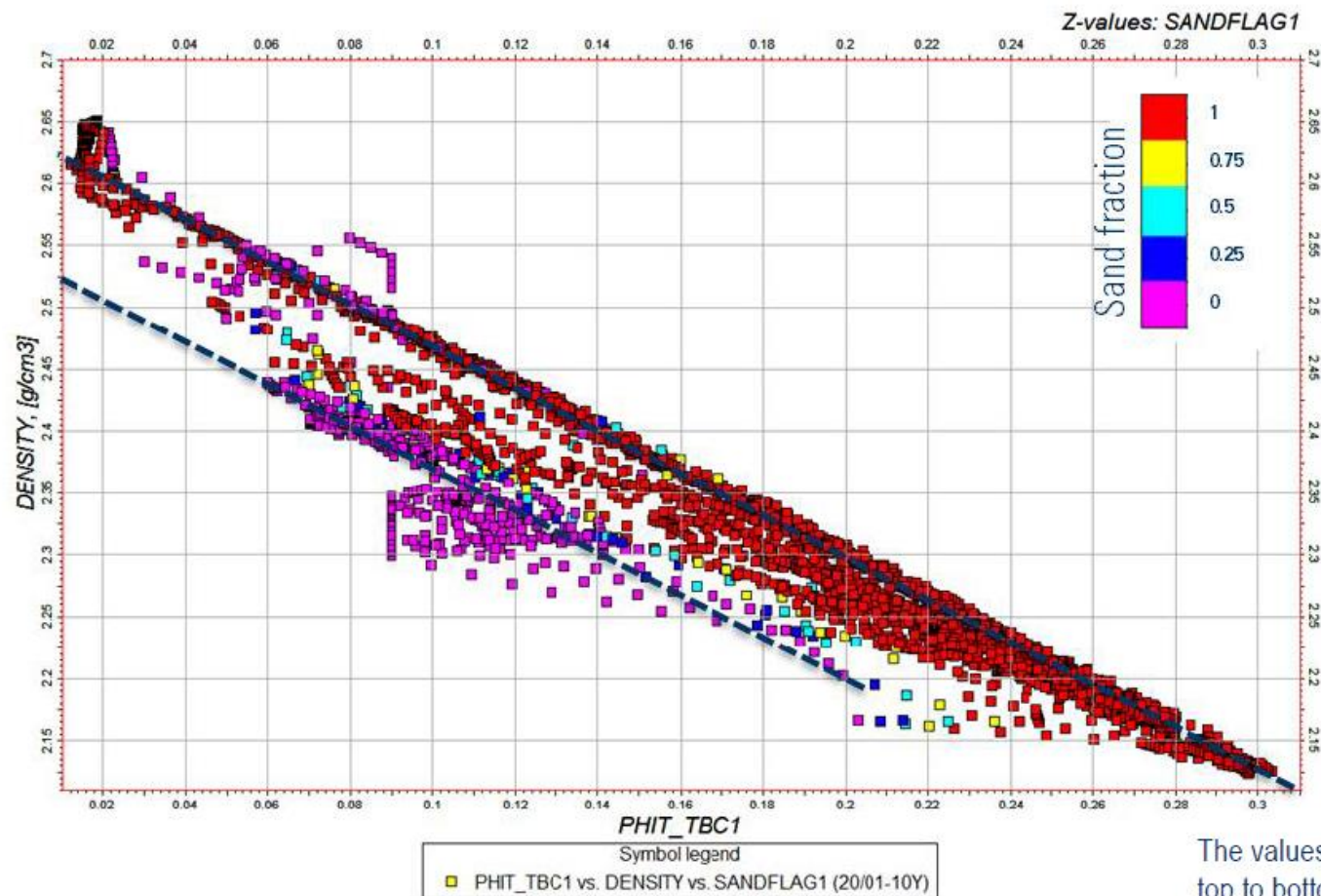
To determine this single value mineral mixing is used



K mineral

Mineral	Bulk modulus (Gpa)	Shear Modulus (Gpa)	Density (g/cm3)
Quartz	36.6	45	2.65
Calcite	76.8	32	2.71
Dolomite	94.9	45	2.87
Clay	25	9	2.6
Muscovite	61.5	41.1	2.79
Feldspar	75.6	25.6	2.63
Halite	24.8	14.9	2.16
Anhydrite	56.1	29.1	2.98
Pyrite	147.4	132.5	4.93
Siderite	123.7	51	3.96

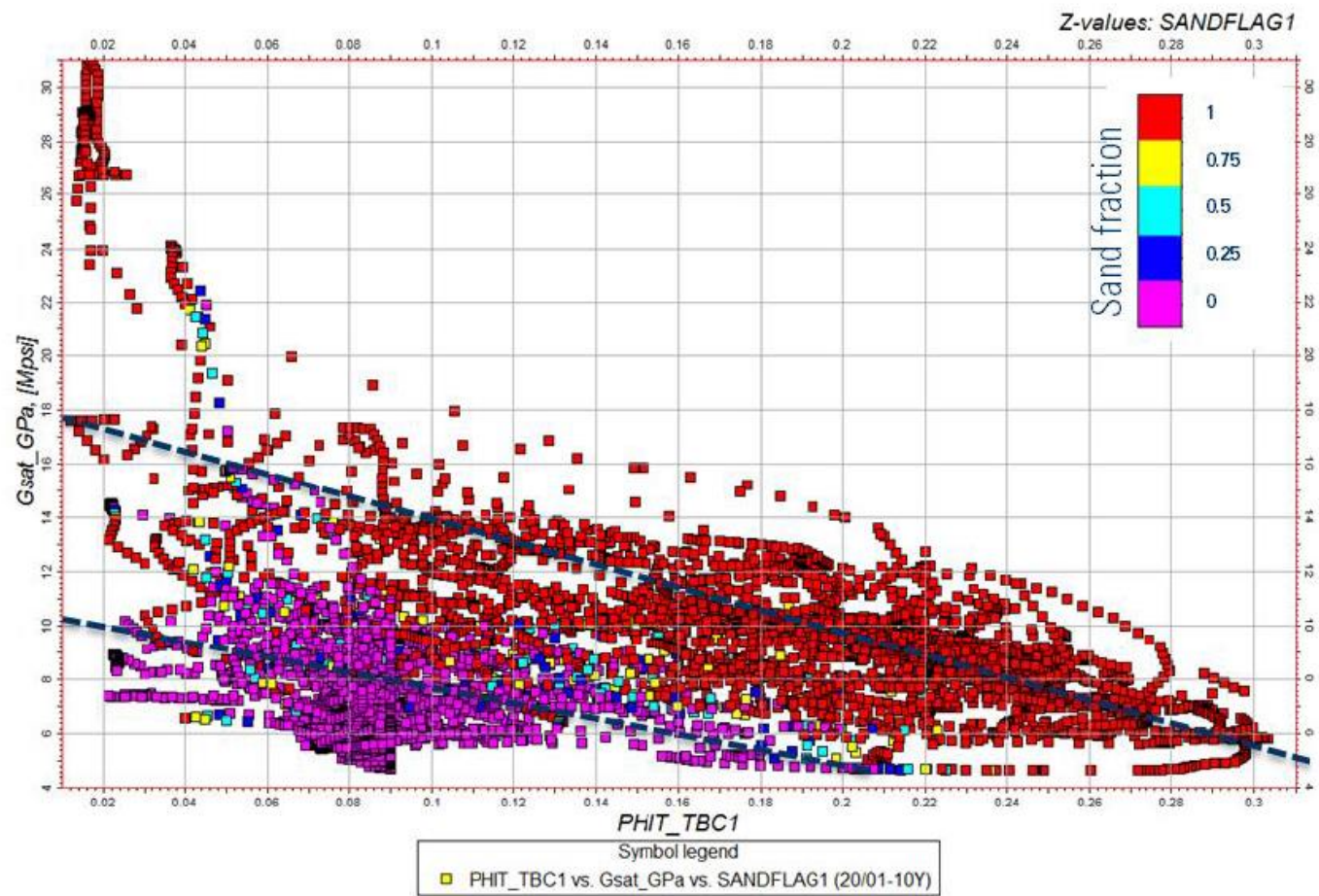
Calibration of mineral density



	Sand	Shale
Km	37	21
Gm	44	7
Density	2.62	2.52

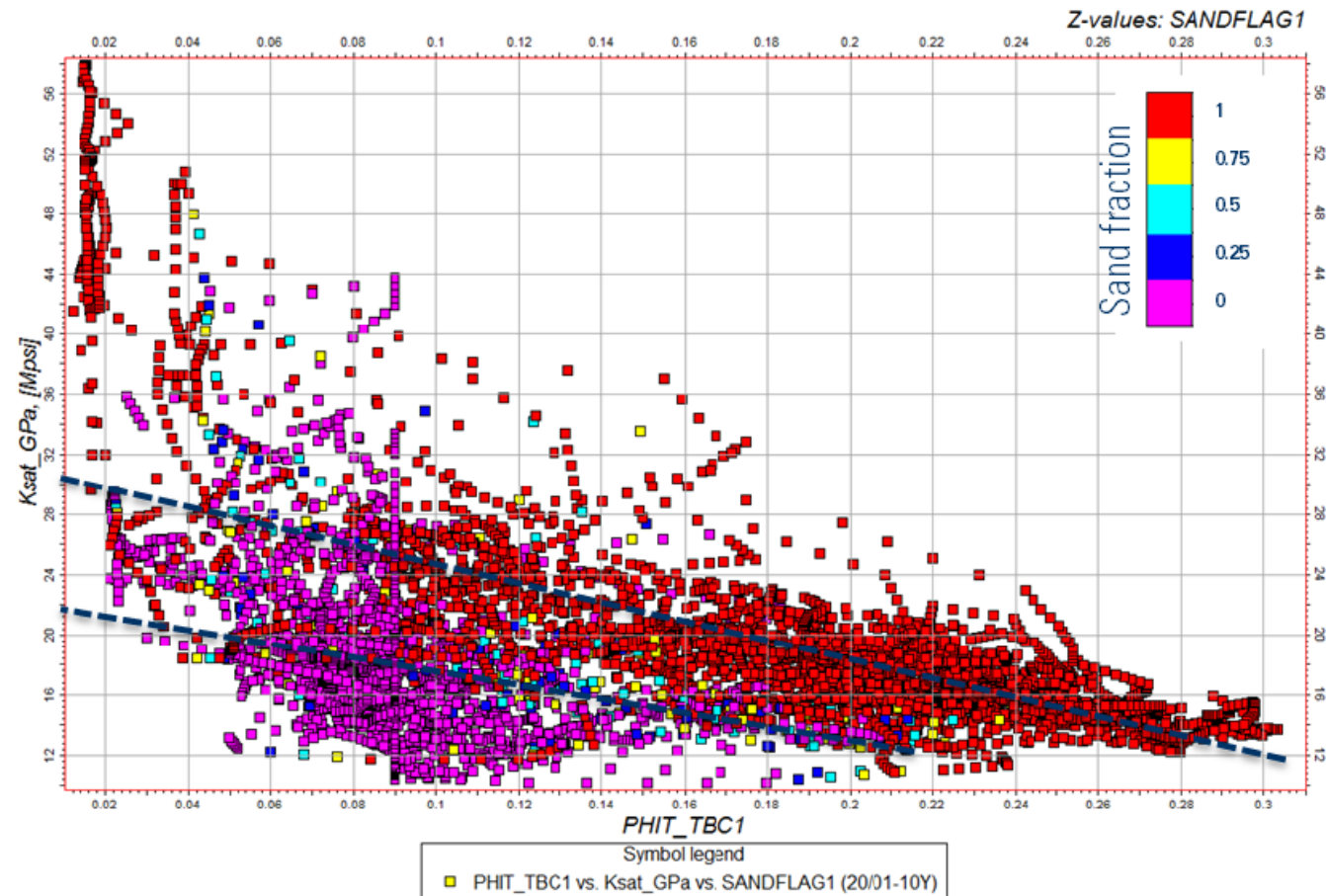
The values are filtered between top to bottom of the reservoir

Calibration of mineral shear modulus (Gm)



	Sand	Shale
Km	37	21
Gm	18	10
Density	2.62	2.52

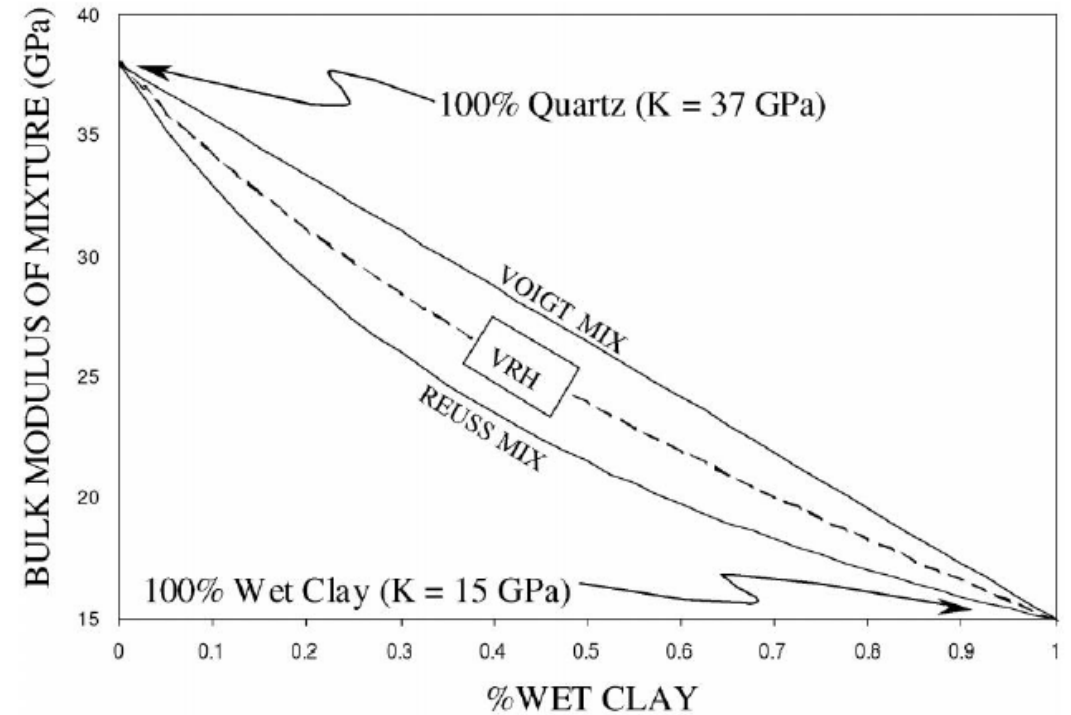
Calibration of mineral bulk modulus (Km)



	Sand	Shale
Km	31	22
Gm	18	10
Density	2.62	2.52

Mineral Mixing - Mixed Bulk and shear moduli of minerals

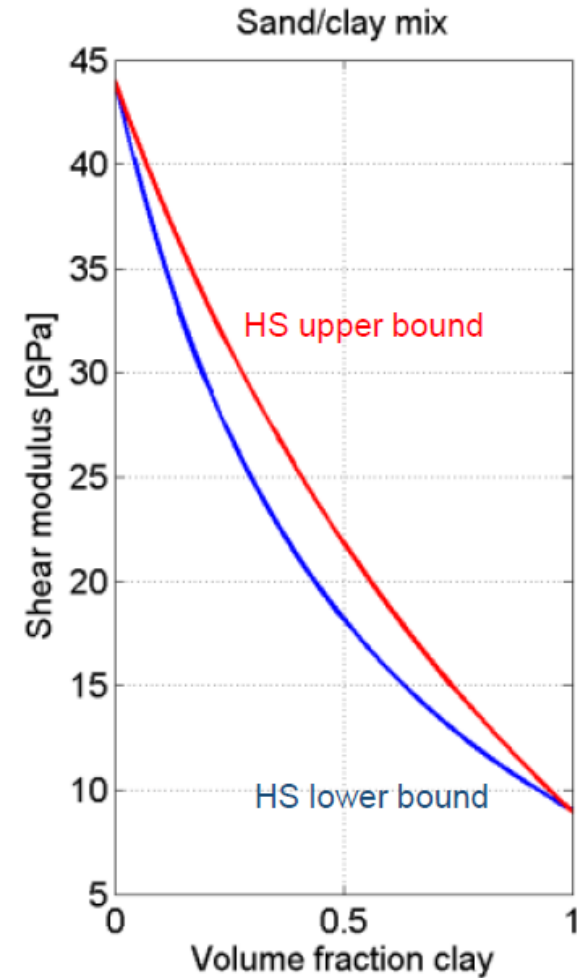
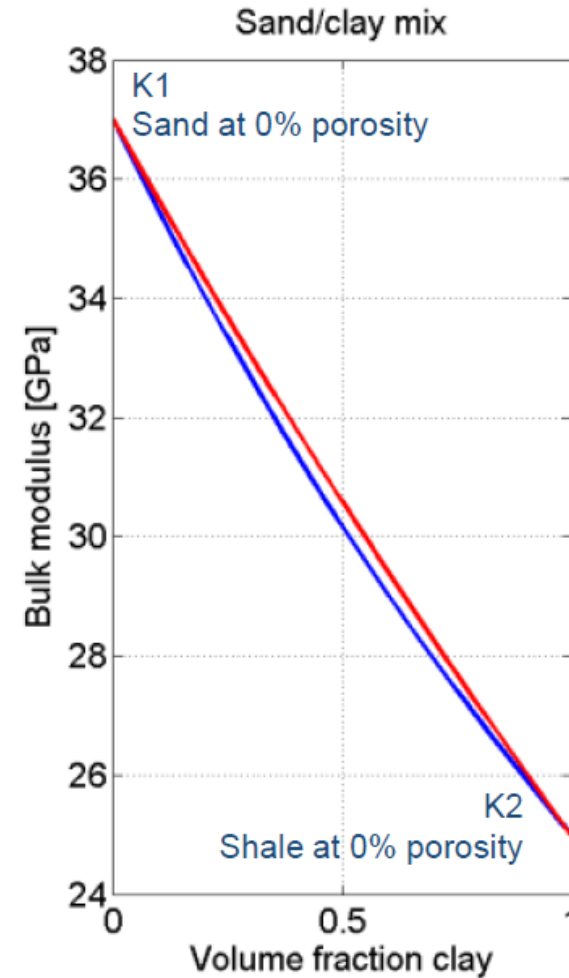
- Once the mineral properties are determined, K_m is calculated via application of Voigt-Reuss-Hill (VRH) averaging of the mineral constituents
- A VRH average is simply the average of the harmonic (Reuss average) and arithmetic means (Voigt average) for the mineral constituents
- Alternatively, Hashin-Shtrikman (HS) averaging may be used



Mineral Mixing for Sand and Shale or Quartz and Clay Mix

Hashin-Shtrikman Average

- The upper bound describes the stiffest a mineral can be based on the volume of clay and the lower the softest. We take an average between the bounds to determine the mixed mineral value.
- Sand at 0% porosity is defined at the stiffest material – K1 and G1
- Shale is defined as the softest material – K2 and G2
- To determine the mixed bulk and shear mineral moduli, an average of the Hashin-Shtrikman bounds is taken.



Sayers & Boer, 2011

Mineral Mixing - Density

Mass balance equation

$$\rho_M = S_A \rho_A + S_B \rho_B$$

↙
mixture density

↘ ↘
densities of components A and B

$S_A, S_B = (1 - S_A)$ - volume fractions or saturations of components A and B