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Fractional Derivative Approach for Pressure Transient data Analysis of Double Porosity Fractal Reservoir with Changing Wellbore Storage

Introduction

This paper presents the estimation of reservoir parameters and its characteristics through readily available dynamic pressure transient test data of oil and gas field by use of fractional derivative in fractal reservoir by fracture/matrix participation with changing wellbore storage effect in geological environment that are not possible by conventional techniques [1, 2]. The analysis of this type of data in reservoir engineering is known as “inverse problem” and one can obtain information about inter-well and vertical permeability distribution in reservoir [3, 4]. This is well known that the inverse problem has no unique solution unless the reservoir model is properly identified by means of diagnostics techniques of derivative analysis by Bourdet et al. in 1989. Fractal geometry plays vital role for heterogeneity characterization in form of dual porosity system [5, 6] given in **figs 1 & 2.** Pressure response is analyzed for flow in connected fracture network with matrix participation.

The aim of pressure transient data interpretation is to establish a reasonable estimate of reservoir properties for better understanding of reservoir behavior. In the reservoirs where majority of fluid flow is through fractures are sensitive to stress regime, its perturbations. Production from the reservoir or injection into the reservoir, cause changes in fluid pressure distribution in the system [7, 8]. This will alter the effective stress, which in turn will affect flow of hydrocarbons from reservoir through fracture network. The geoscientist such as geologists, petrophysicists, geophysicists, production and reservoir engineers have brought forward more reasonable expectations by sharing their knowledge to integrate various type of data in drilled wells to finding heterogeneity [9].

The computer aided matching techniques for measured pressure data and its calculated derivative and ratio with pressure is used to estimate reservoir properties to avoid human errors. This transient test data provide clues about the nature of fracture and dual porosity network within reservoir and help in extrapolating fracture from individual wells to between wells and ultimately in whole reservoir. Simulated pressure derivative show different characteristics with different fracture patterns configurations [10, 11]. Permeability estimated from transient test data is utilized to generate 3D model by combining porosity derived from well log and seismic data with geo-statistical techniques.

Fractional calculus is the field of mathematical analysis which deals with investigation and application of derivatives and integrals of real roots. It is an old topic since its development started from G. W. Leibniz and L. Euler. In recent years the interest for fractional calculus has been stimulated by its wide survey of applications. We begin with the definition of a fractional derivative and its application since they are used in the formulation of the problems. Several definitions of a fractional derivative and integral have been proposed. The most frequently used definition of a fractional derivative of order $\alpha > 0$ is the Riemann-Liouville definition, which is straightforward generalization to non-integer values of Cauchy formula. The Riemann-Liouville fractional derivative is defined as [1]

$$D^{\alpha} f(t) = \begin{cases} \dfrac{d^n}{dt^n}\left[\dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t \dfrac{f(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau\right], n-1 < \alpha < n \\ \dfrac{d^n}{dt^n} f(t), \alpha = n \end{cases} \tag{1}$$

Where, Γ is the Gamma function, n is positive integer such that n-1< α < n and $\alpha > 0$.
An alternative definition of the fractional derivative was proposed by Caputo [1]

$$D^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t (t-\tau)^{n-\alpha-1} \dfrac{d^n f(\tau)}{d\tau^n} d\tau, n-1 < \alpha < n \\ \dfrac{d^n}{dt^n} f(t), \alpha = n \end{cases} \tag{2}$$

Mathematical Formulation of the Physical Problem

It is important to emphasize that what seems to be really interesting in studying fractals via fractional calculus, are the non-integer physical dimensions that arise dealing with both fractional operators and fractals sets. The mathematical formulation of the pressure transient equation for fluid flow through fractal reservoir where the fracture network is largely divided by backbone fractures and fractal fracture loop has been considered. Change and Yortsos considered a Euclidean matrix within which the fracture network is embedded, obtaining a generalization of Warren and Root model for a Euclidean geometry for a dual porosity system as follows

$$\omega r_D^{\theta} \frac{\partial^{\gamma} P_{fD}}{\partial t_D^{\gamma}} = \frac{\partial^2 P_{fD}}{\partial r_D^2} + \frac{\beta}{r_D}\frac{\partial P_{fD}}{\partial r_D} + \frac{\lambda r_D^{\theta}}{3}\left(\frac{\partial P_{mD}}{\partial z_D}\right) \tag{3}$$

Where d and d_f are Euclidean and mass fractal dimensions, respectively, $\beta = d_f - \theta - 1$, with θ being the conductivity index, $0 \le \gamma < 1$from the definition of fractional diffusion equation of order, γ and the storativity ratio,ω and interporosity flow parameter between fracture and matrix, λ is given by, $\omega = c_{tf}/(c_{tf} + \phi_m c_{tm})$, $\lambda = \dfrac{12k_m}{m_f}\left(\dfrac{r_w^{\theta+2}}{h_m^2}\right)$,$\gamma = 2/d_w$, and $0 \le \gamma < 1$ from the definition of fractional diffusion equation of order γ, $\beta = d_f - \theta - 1$. This equation reduces to standard diffusion equation, when $\gamma = 1$, $\theta = 0$, and d_f =1, 2, 3 for Euclidean dimension.
The matrix expression was given as follows
Taking the Laplace transform of the governing partial differential equation (3) yields

$$\frac{d^2 \overline{P}_{fD}}{dr_D^2} + \frac{\beta}{r_D}.\frac{d\overline{P}_{fD}}{dr_D} - s^{\gamma} r_D^{\theta}\left[\omega + \frac{\lambda}{3s^{\gamma}}\frac{d\overline{P}_{mD}}{dz_D}\right]\overline{P}_{fD}(s) = 0 \tag{4}$$

It has been found from earlier studies that the contribution of the matrix blocks in a naturally fractured reservoir is to supply fluid to the fracture network. The fractures then transmit the fluid to the wellbore. Only the fracture network will be treated as fractal since fluid flow only takes place in the fracture network. The 1- D matrix transfer differential has been solved by Olarewaju [12] and the term $(d\overline{P}_{mD}/dz_D)$ for slab type of block model is given as

$$\frac{d\overline{P}_{mD}}{dz_D} = \sqrt{\frac{3s(1-\omega)}{\lambda}} \tanh\left(\sqrt{\frac{3s(1-\omega)}{\lambda}}\right)\overline{P}_{fD} \tag{5}$$

Substituting equation (5) into equation (4) for the matrix term, we get

$$\frac{d^2 \overline{P}_{fD}}{dr_D^2} + \frac{\beta}{r_D}.\frac{d\overline{P}_{fD}}{dr_D} - s^{\gamma} r_D^{\theta}(f_s)\overline{P}_{fD}(s) = 0 \tag{6}$$

where, $f_s = \omega + (1-\omega)\left[\sqrt{\dfrac{\lambda}{3s^{2\gamma-1}(1-\omega)}}\tanh a\right]$, and $a = \left(\sqrt{\dfrac{3s(1-\omega)}{\lambda}}\right)$ (7)

The initial and inner boundary conditions are given as

$$P_{fD}(r_D,0)=0 \quad (8)$$

$$P_{wD} = \left[P_{fD} - Sr_D^{\beta}\frac{\partial P_{fD}}{\partial r_D}\right]_{r_D=1} \quad (9)$$

$$C_D\frac{dP_{wD}}{dt_D} - \left[r_D^{\beta}\frac{\partial P_{fD}}{\partial r_D}\right]_{r_D=1} = 1 \quad (10)$$

The outer boundary condition is that of an infinite acting reservoir given by

$$P_{fD}(r_D,t_D)\to 0, \text{ as } r_D\to\infty \quad (11)$$

The dimensionless phase redistribution was modeled as a changing wellbore storage phenomenon. The pressure function $P_{\Phi D}$ has the following properties

$$\lim_{t\to 0} P_{\phi D} = 0 \quad (12)$$

$$\lim_{t\to\infty} P_{\phi D} = C_{\phi D} \text{ ,a constant} \quad (13)$$

$$\lim_{t\to\infty}(dP_{\phi D}/dt_D) = 0 \quad (14)$$

Application of increasing/decreasing wellbore storage model to field data was first used by Fair as exponential form for changing storage pressure function as

$$P_{\phi D} = C_{\phi D}\left(1-e^{-t_d/\alpha_D}\right) \quad (15)$$

Solution of the Physical Problem

Also applying the Laplace transformation to the boundary conditions from (8) to (15), and solving the equation (6) with initial and boundary conditions, we obtain the solution as

$$\overline{P}_{fD}(z) = \frac{K_\nu\left(\frac{2}{\theta+2}\{\sqrt{s^\gamma f_s}\}\right)}{s\left[(\sqrt{s^\gamma f_s})K_{1-\nu}\left(\frac{2}{\theta+2}\{\sqrt{s^\gamma f_s}\}\right)\right]} \quad (16)$$

$$\overline{P}_{\phi D} = \left[C_{\phi D}/z - C_{\phi D}/(z+\alpha_D^{-1})\right] \quad (17)$$

Where $\nu = (1-\beta)/(\theta+2)$

The general solution for wellbore pressure ($\overline{P}_{wD}$) in terms of $\overline{P}_{fD}$ and $\overline{P}_{\phi D}$ in Laplace space is given as

$$\overline{P}_{wD}(z) = \frac{\left[s\overline{P}_{fD}+S\right]\left[1+C_D s^2\overline{P}_{\phi D}\right]}{s\left[1+C_{mD}s^2+C_D s\{s\overline{P}_{fD}+S\}\right]} \quad (18)$$

Where, C_{mD} is dimensionless mass in wellbore

The inverse Laplace transform of equation (18) is calculated numerically to find P_{wD} by using well known Stehfest algorithm. If the matrix blocks are cubes or spheres, then the inter-porosity flow is 3-D and λ is given as, $\lambda = \dfrac{60k_m}{x_m^2 k_f} r_w^2$, where x_m is the length of a side of the cubic block, or the diameter of the spherical block. If the matrix blocks are long cylinders, then the inter-porosity flow is 2D and λ is given by, $\lambda = \frac{32k_m}{x_m^2 k_f} r_w^2$, where x_m is now the diameter of the cylindrical block. If the matrix blocks are slabs overlaying each other with fractures in between, then the inter-porosity flow is 1D, and λ is given by, $\lambda = \frac{12k_m}{h_f^2 k_f} r_w^2$, where h_f is the height of the secondary porosity of slab. The Dimensionless pressure, its derivative and the ratio of pressure derivative with pressure plots are given in **figures 3-10** for different combination of the reservoir parameters for dual porosity fractal reservoir with changing wellbore storage parameters.

Conclusions

This paper describes the transient pressure response of naturally fractured reservoir with changing wellbore storage effect during the transient and pseudo-steady state flow period by using fractional calculus in fractal reservoir. In reservoir where fractures are much more permeable than matrix, the derivatives of a pressure transient well test are a good indication of the underlying fracture network. The various reservoir parameters including shape of matrix blocks are estimated by using the dynamic pressure transient test data. The effect of momentum on the wellbore pressure response has also been considered. It is noted that fracture permeability in a fractured reservoir is very sensitive to stress change due to pressure depletion or injection. We have matched pressure, its derivative & their ratio data with regression analysis technique of some well (**figs.11 & 12**). The acquisition of these data and their continued evaluation at unknown locations sounds reservoir management to develop the field and implement applicable improved and Enhances oil recovery schemes.

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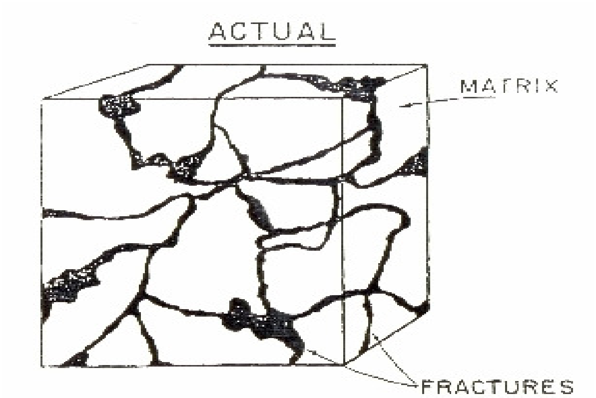

Fig.1 Schematic diagram of naturally fractured reservoir in actual.

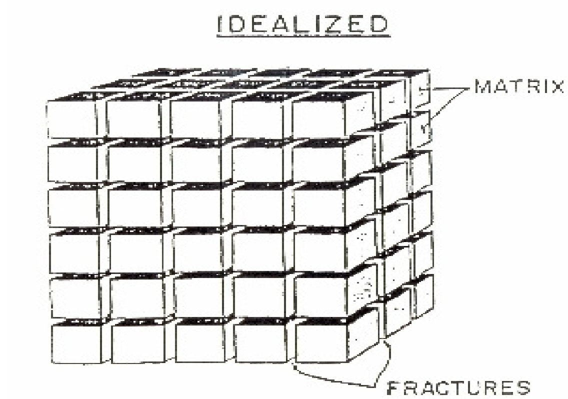

Fig. 2 Schematic diagram of Modeled naturally fractured reservoir.

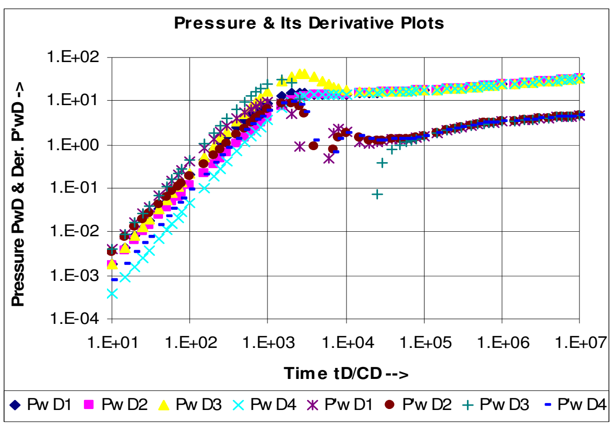

Fig.3 P_{wD} & P'_{wD} for, $C_D=10^2$, $C_{mD}=10^6$, S=2, $S_F=0$, $d_f=1.9$, $d_w=2.1$ $C_{\Phi D}=0, 1, 10, 10^2$, $\alpha_D=0, 25, 250, 2.5\text{x}10^3$, $\omega=10^{-3}$, $\lambda=10^{-5}$, $\theta=0.2$

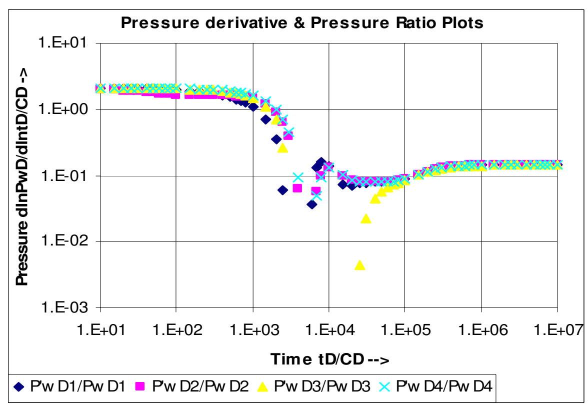

Fig. 4 P'_{wD}/P_{wD} for, $C_D=10^2$, $C_{mD}=10^6$, S=2, $S_F=0$, $d_f=1.9$, $d_w=2.1$, $C_{\Phi D}=0, 1, 10, 10^2$, $\alpha_D=0, 25, 250, 2500$, $\omega=10^{-3}$, $\lambda=10^{-5}$, $\theta=0.2$

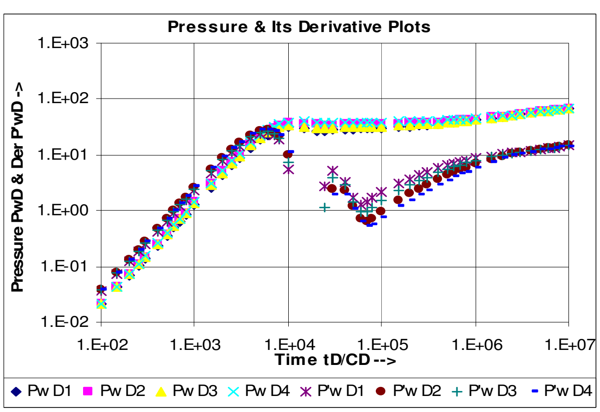

Fig.5 P_{wD} & P'_{wD} for, $C_D=10^2$, $C_{mD}=10^6$, S=2, $S_F=0.5,1,2,3$, $d_f=1.9$, $d_w=2.1$, $C_{\Phi D}=1$, $\alpha_D=25$, $\omega=10^{-3}$, $\lambda=10^{-5}$, $\theta=0.2$

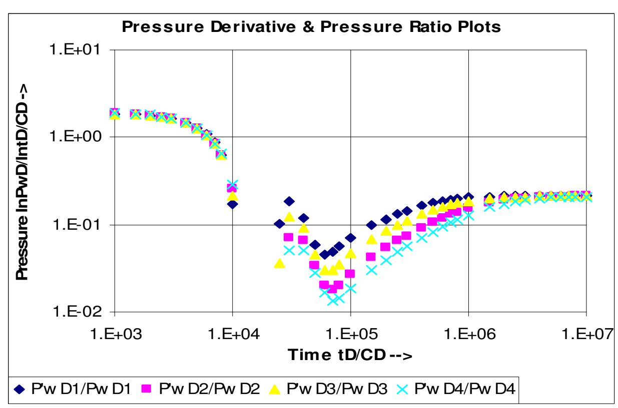

Fig. 6 P'_{wD}/P_{wD} for, $C_D=10^2$, $C_{mD}=10^6$, S=0, $S_F=0.5, 1, 2, 3$, $d_f=1.9$, $d_w=2.21$, $C_{\Phi D}=1$, $\alpha_D=25$, $\omega=10^{-3}$, $\lambda=10^{-5}$, $\theta=0.4$

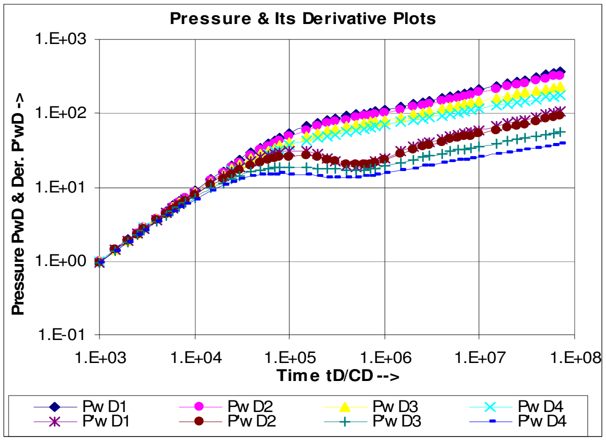

Fig. 7 P_{wD} & P'_{wD} for, $C_D=10^2$, $C_{mD}=0$, $S=0$, $S_F=2$, $d_f=1.7$, $d_w=2, 2.21,2.5, 2.7$, $C_{\Phi D}=1$, $\alpha_D=25$, $\omega=0.1$, $\lambda=10^{-5}$, $\theta=0.4$

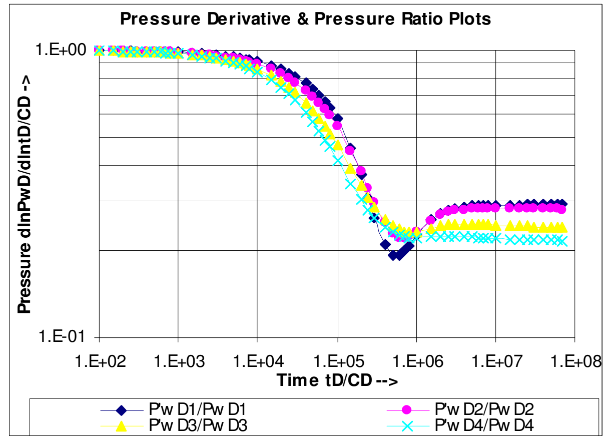

Fig. 8 P'_{wD}/P_{wD} for, $C_D=10^3$, $C_{mD}=0$, $S=5$, $S_F=2$, $d_f=1.7$, $d_w=2, 2.21, 2.5, 2.7$ $C_{\Phi D}=0$, $\alpha_D=0$, $\omega=0.1$, $\lambda=10^{-5}$, $\theta=0.4$

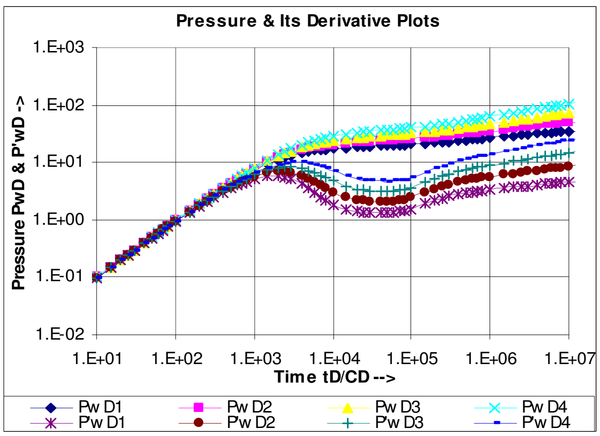

Fig.9 P_{wD} & P'_{wD} for, $C_D=10^2$, $C_{mD}=0$, $S=5$, $S_F=0$, $d_f=1.9$, $d_w=2.2$, $C_{\Phi D}=0$, $\alpha_D=0$, $\omega=10^{-3}$, $\lambda=10^{-5}$, $\theta=0.2, 0.3, 0.4, 0.5$

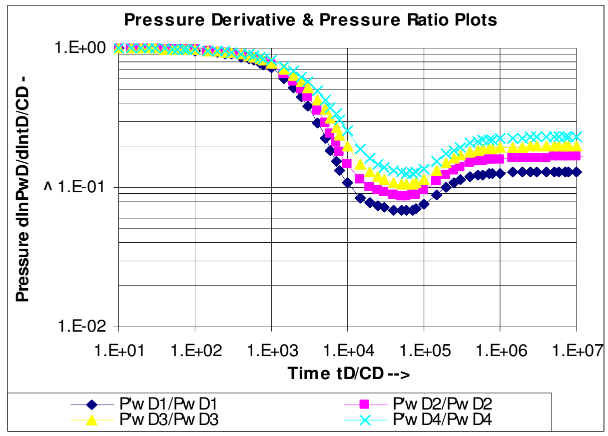

Fig. 10 P'_{wD}/P_{wD} for, $C_D=10^2$, $C_{mD}=0$, $S=5$, $S_F=0$, $d_f=1.9$, $d_w=2.1$, $C_{\Phi D}=0$, $\alpha_D=0$, $\omega=10^{-3}$, $\lambda=10^{-5}$, $\theta=0.2, 0.3, 0.4, 0.5$

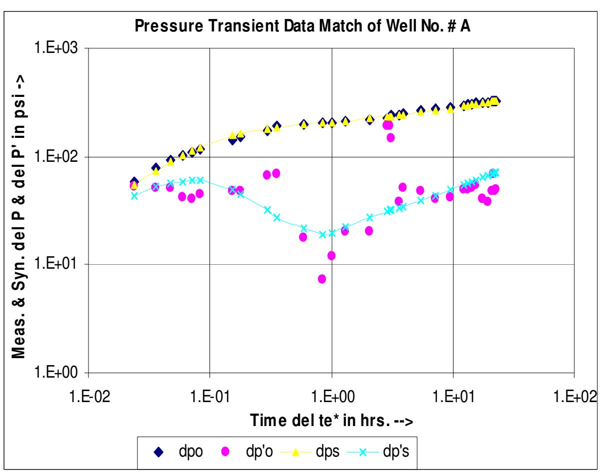

Fig. 11 del P & del P' for, $C_D=973$, $S=-0.56$, $d_f=2$, $d_w=2$, $C_{\Phi D}=0$, $\alpha_D=0$, $\omega=0.24$, $\lambda=8.710^{-6}$, K=218.068

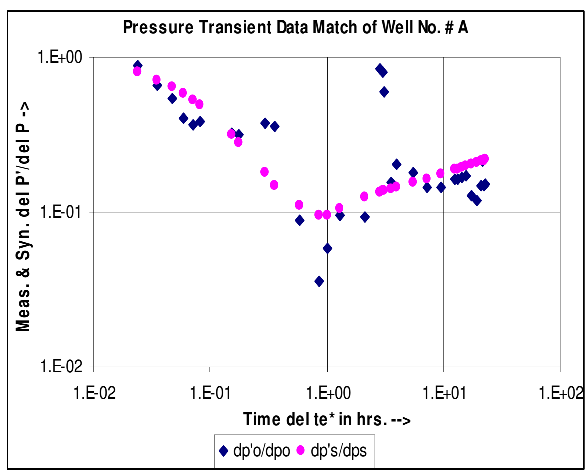

Fig. 12 del P'/del P for, $C_D=973$, $S=-0.56$, $d_f=2$, $d_w=2$, $C_{\Phi D}=0$, $\alpha_D=0$, $\omega=0.24$, $\lambda=8.710^{-6}$, K=218.068